

My comments focus on
“the discussion section”

Did *neriage* happen ?
*kneading or polishing ideas through
discussion*

webinar

Wednesday 21 October 2020

7:00pm – 8:30pm AEDT, 5:00pm – 6:30pm JST

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We need to understand the background information

- * 1. **Composite class:** mixture of Grade 3 and Grade 4
- * 2. According to Felicity sensei, they did 2digit(11~19) \times 1 digit at previous lesson.
- * 3. The class **has not done** multiples of 6, therefore **43×6 is a challenging task for students.**
- * So that “complicated strategy” (halving 6 & doubling the product of 43×3) is a beautiful solution by them.
- * This is a beautiful evidence to prove **students are truly independent thinkers**

Handwritten work showing the calculation of 43×6 using a strategy of doubling 43×3 . The work is split into two panels. The left panel shows 43 and 3 with a multiplication sign and a horizontal line. The right panel shows 129 and 129 with a plus sign and a horizontal line, and 258 below it.

Ari's strategy

when we talk about the lesson, we always need to consider the goal of the lesson.

Goal of the lesson :

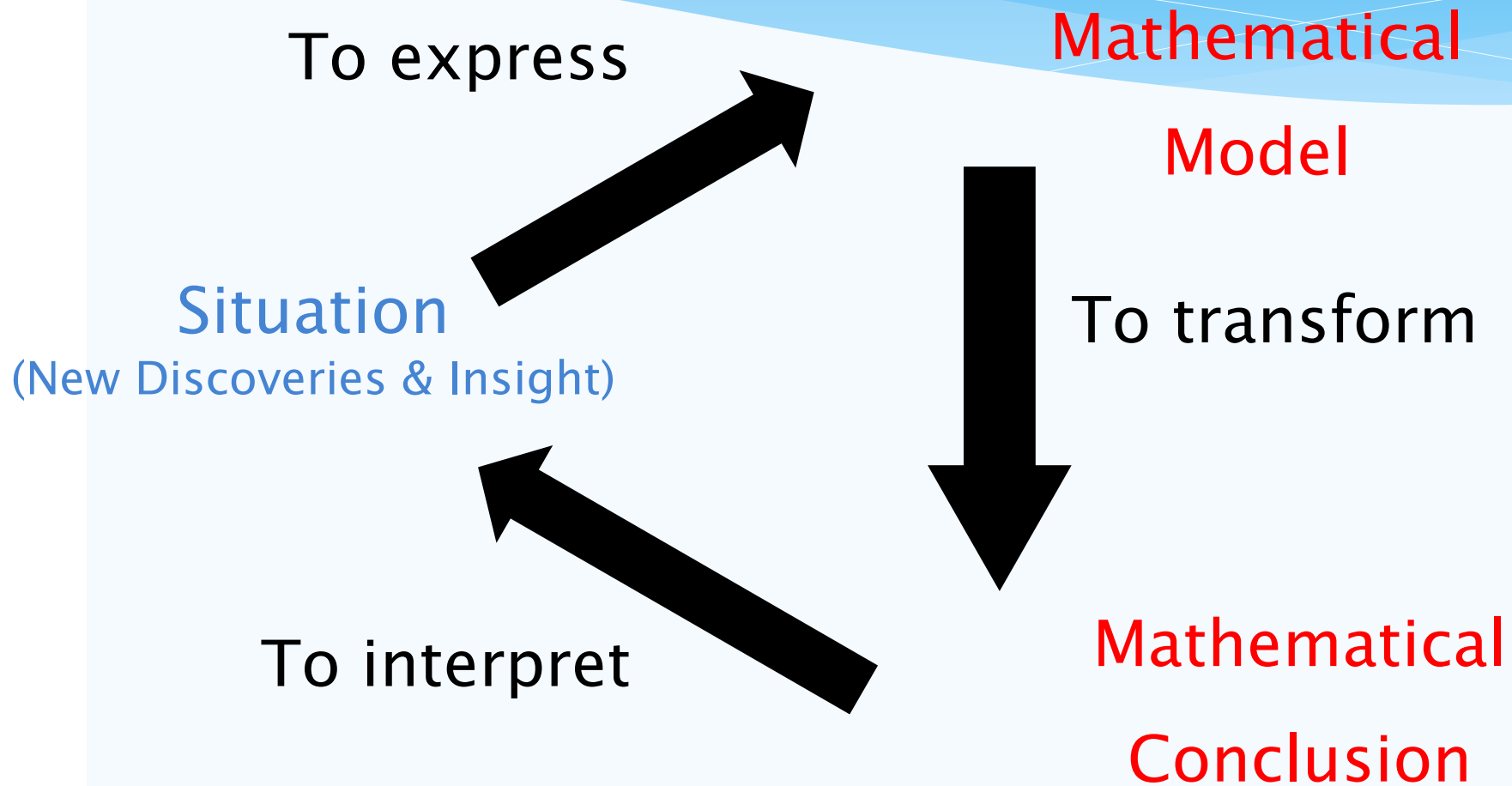
To develop and extend the use of efficient strategies

To recognize that some strategies are more efficient than others

Did discussion move forward to the goal?

- * We need a bit more clearer goal
- * What does it mean by “efficient strategies” and “some strategies are more efficient than others”

I am going to use the framework below:
Scheme of Use of Mathematical Expressions:
Process of mathematical problem solving



The words **problem** is located between the situation and mathematical model

Scheme of Use of Mathematical Expressions: Process of mathematical problem solving



Situation

(New Discoveries & Insight)

A new theme park, birthday party.

To express

A new theme park has opened in Geelong and you are going there for your birthday party. You and five friends are going to the party. Tickets cost \$43 each. How much will it cost altogether?

Mathematical Model

To transform

Mathematical Conclusion

To interpret

It is better to put “ 43×6 ” and “ $43+43+43+43+43+43$ ” on the white board at the lesson

Scheme of Use of Mathematical Expressions: Process of mathematical problem solving



Situation

(New Discoveries & Insight)

A new theme park has opened in Geelong and you are going there for your birthday party. You and five friends are going to the party. Tickets cost \$43 each. How much will it cost altogether?

To express

$$43 \times 6$$

Mathematical

$$43+43+43+43+43+43$$

Model

To transform

Mathematical

$$258$$

Conclusion

To interpret

Total cost \$258 for six children

A new theme park, birthday party.

240
18
groups of

S1

Repeated addition

$$40 + 40 + 40 + 40 + 40 + 40 = 240$$

$$3 + 3 + 3 + 3 + 3 + 3 = 18$$

$$240 + 18 = 258$$

S2: Jack: repeated addition

4 x 6 = 24 then

added a zero on

240

$$6 \times 3 = 18$$

$$240 + 18 = 258$$

S3: Ryan: split multiplication

$$\begin{array}{r} \$43 \\ \times 3 \\ \hline \end{array}$$

S4

$$\begin{array}{r} \$129 \\ + \$129 \\ \hline \$258 \end{array}$$

$$\begin{array}{r} \$40 \\ \times 6 \\ \hline \$240 \end{array}$$

Vertical multiplication with Split Strategy.

$$\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$$

S5



Situation

(New Discoveries & Insight)

A new theme park, birthday party.

A new theme park has opened in Geelong and you are going there for your birthday party. You and five friends are going to the party. Tickets cost \$43 each. How much will it cost altogether?

To express



Mathematical Model

$$43 \times 6$$

To transform



Mathematical Conclusion

$$258$$

To interpret

Total \$258 for six children



Strategies (S1 to S5) are all located at the process of "To transform".
I need another 30 minutes to explain why Strategy 1 "Group of" is located at two places.

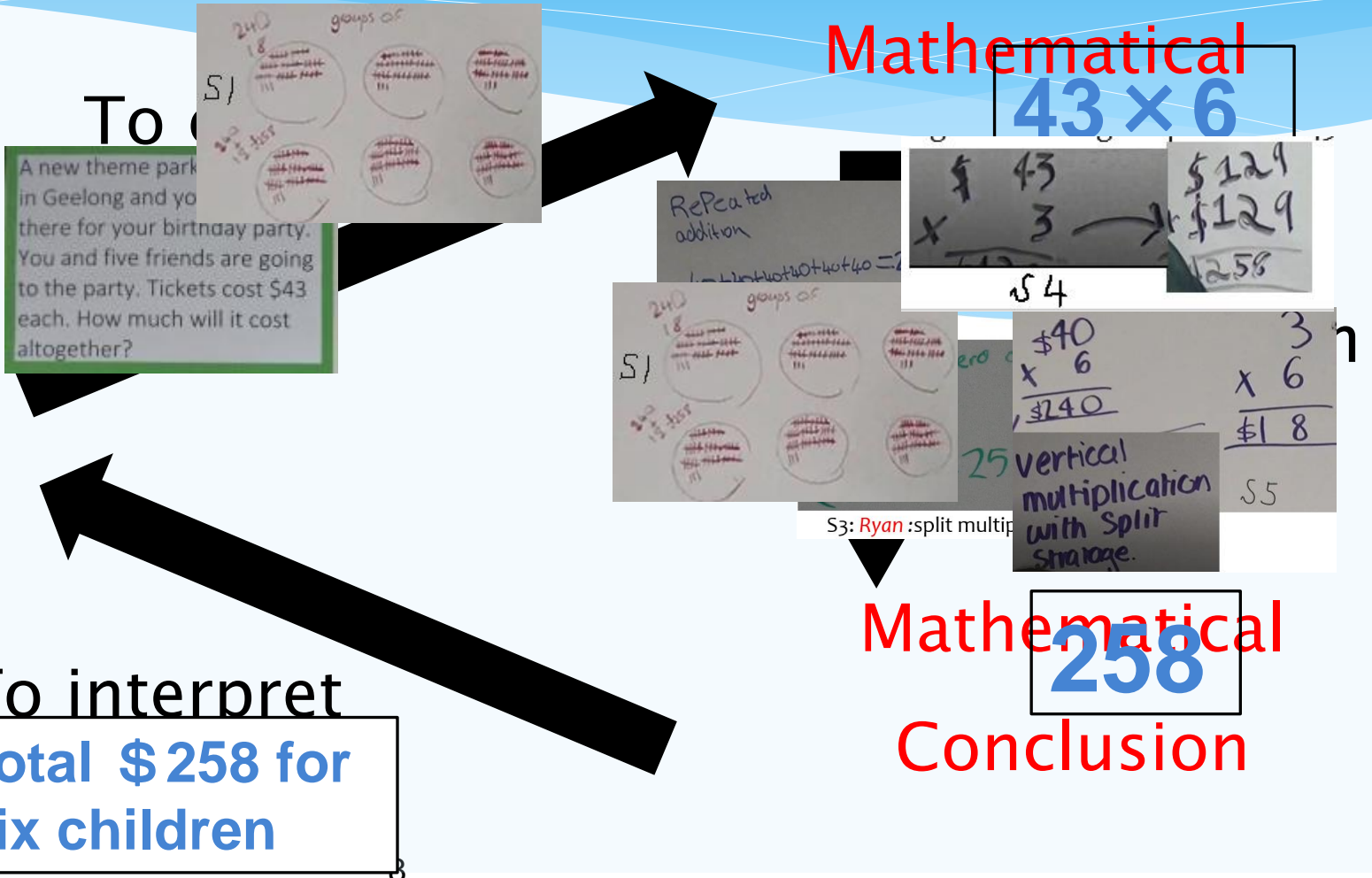
Scheme of Use of Mathematical Expressions: Process of mathematical problem solving



Situation

(New Discoveries & Insight)

A new theme park, birthday party.



“efficient strategies” and “some strategies are more efficient than others”

Did discussion move forward to the goal?

- * Strategy 1 to Strategy 5 are all located at the transforming place
- * That is the calculating process
- * Algorithms !
- * We should not forget the lesson started from daily life situation
- * Did discussion move forward to think about “some strategies are more efficient than others” ?
- * Was discussion beyond Show and Tell?
- * Did *Nariage* happen at discussion section?

What is the neriage ?

It should be an actualization of ZPD

- * Neriage is beyond “Show and Tell”
- * Although rather a classic citation, **neriage should be an actualization of Vygotsky’s zone of proximal development**
- * Therefore, during neriage, **teachers must see the students’ potential as high as possible** and **help them move toward their potential from what they came up on their own**, i.e., what they can do without help.

The first *Neriage* occurred in six minutes [27:18] to [33:36] and was discussed by students in Elise's diagram. It is an actualization of Communities of Inquiry (see Susie sensei's slide)



S4: Elise? Why did you do "groups of"?

Elise: Because it was just one of the strategies that I selected. Yeah, that's why I did it.

S4: But it's not the most efficient.

Elise: I know it's not, but I just wanted to keep it straight forward.

Max: Did you use the tally marks...

Elise. Yes, I did, because that's more efficient than using just, like, strokes.

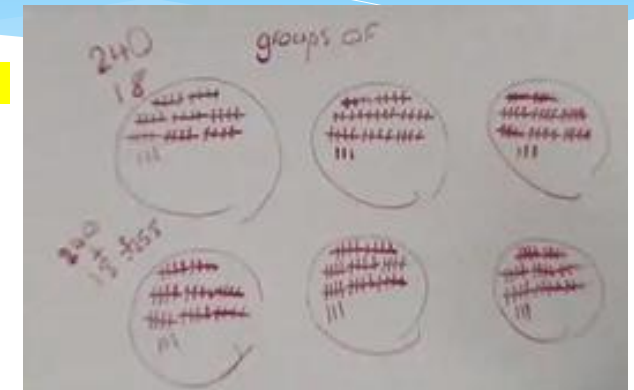
Elana: Why did you use six groups?

Elise: Because there's six people.

Jack: Elise, why didn't you do just one group and times it by six, because it would've been much quicker than this way.

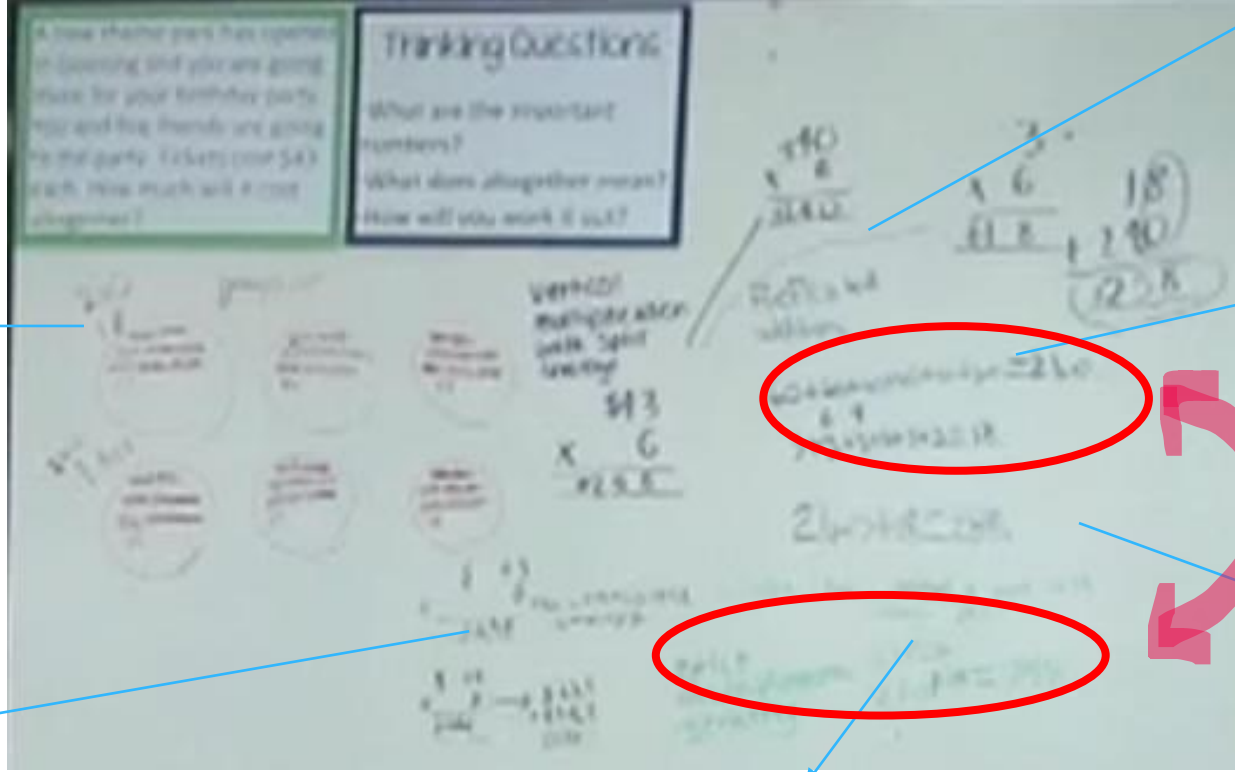
Elise: Well, I didn't think to do that, so...

T: It might be difficult for some people to do that, too. Jack, sometimes people need to see something visually, and be able to use that and count out things like their thought on the board.



Another Neriage is identified by Wanty sensei

From Wanty sensei PP **“Comparing and contrasting”**



S1: Groups of

S5: Vertical multiplication with a split strategy

S2: repeated addition

$$40+40+40+40+40+40 = 240$$

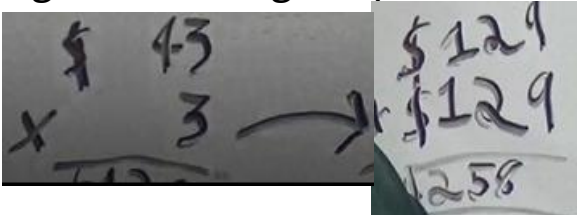
$$3+3+3+3+3+3 = 18$$

$$240+18 = 258$$

24+0??

Why did you add the zero?

S4: “complicated strategy”
(halving 6 & doubling the product of 43x3)



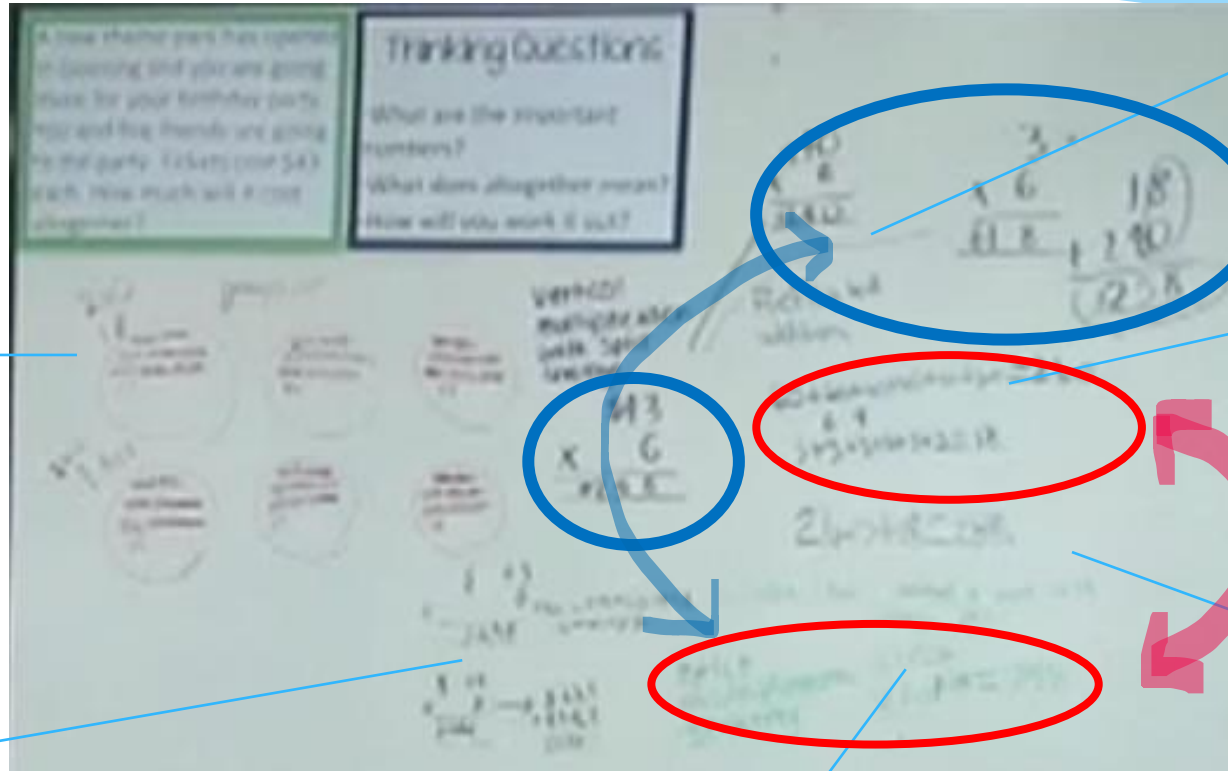
S3: split multiplication strategy

$$4 \times 6 = 24 \text{ then I added a zero on the answer } 240.$$

$$6 \times 3 = 18$$

$$240 + 18 = 258$$

Another *neriage*? Another possible place for discussing on efficiency: that is between S3 and S5

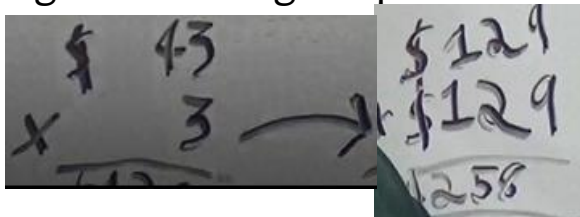


S1: Groups of

S5: Vertical multiplication with a split strategy

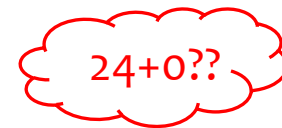
S2: repeated addition
 $40+40+40+40+40+40 = 240$
 $3+3+3+3+3+3 = 18$
 $240+18 = 258$

S4: "complicated strategy"
 (halving 6 & doubling the product of 43×3)



S3: split multiplication strategy

$4 \times 6 = 24$ then I added a zero on the answer 240.
 $6 \times 3 = 18$
 $240 + 18 = 258$



Why did you add the zero?

Neriage (a colloquial term):

kneading or polishing ideas through discussion

*To do this *neriage*, we may need an additional component

we may need an additional component between 5a(split strategy) and 6(vertical multiplication)

5a

$$40 \times 6 = 240$$
$$3 \times 6 = 18$$
$$\begin{array}{r} + 240 \\ 18 \\ \hline \$258 \end{array}$$

multiplication
split strategy



6

$$\begin{array}{r} 43 \\ \times 6 \\ \hline \$258 \end{array}$$

vertical
multiplication

5b or

$$\begin{array}{r} 40 \\ \times 6 \\ \hline 240 \end{array}$$
$$\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$$
$$\begin{array}{r} 240 \\ + 18 \\ \hline \$258 \end{array}$$

That is the longer version of vertical multiplication

5a

$$40 \times 6 = 240$$
$$3 \times 6 = 18$$

$$\begin{array}{r} + 240 \\ 18 \\ \hline \$258 \end{array}$$

multiplication
split strategy

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$$

6

$$\begin{array}{r} 43 \\ \times 6 \\ \hline \$258 \end{array}$$

vertical
multiplication

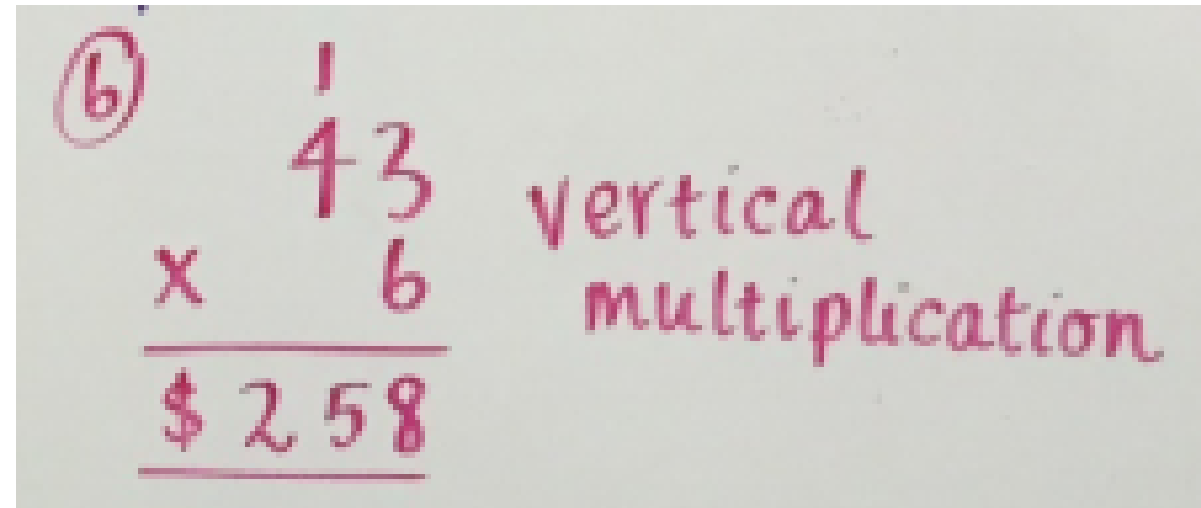
5b or

$$\begin{array}{r} 40 \\ \times 6 \\ \hline 240 \end{array} \quad \begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array} \quad \begin{array}{r} 240 \\ + 18 \\ \hline \$258 \end{array}$$

Longer version of vertical multiplication

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$$

Shorter version of vertical multiplication



A photograph of a hand-drawn multiplication problem on a light-colored surface. The problem is circled in red and labeled 'vertical multiplication' in red. The numbers are written in red ink. The multiplication is 43 times 6, with a horizontal line under the 6, and the result is 258, with a horizontal line under the 258. The result is preceded by a dollar sign.

$$\textcircled{6} \begin{array}{r} 43 \\ \times 6 \\ \hline \$ 258 \end{array} \text{ vertical multiplication}$$

$$\begin{array}{r} \$ 43 \\ \times 6 \\ \hline 1258 \end{array}$$

$$\begin{array}{r} \$40 \\ \times 6 \\ \hline \$240 \\ + 18 \\ \hline 258 \end{array}$$

Repeated addition

$$40 + 40 + 40 + 40 + 40 + 40 = 240$$

$$\begin{array}{r} 69 \\ 3+3+3+3+3+3 = 18 \end{array}$$

$$240 + 18 = 258$$

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$$

Vertical multiplication with split strategy.

split multiplication strategy

$$4 \times 6 = 24 \text{ then } 240$$

$$6 \times 9 = 54$$

$$240 + 54 = 294$$

24+0??
Why did you add the zero?

Through introducing the long version of multiplication strategy, strategies given by students can be integrated into the long version of vertical multiplication

$$\begin{array}{r} \$ 43 \\ \times 6 \\ \hline 1258 \end{array}$$

$$\begin{array}{r} \$40 \\ \times 6 \\ \hline \$240 \end{array}$$

$$\begin{array}{r} 30 \\ \times 6 \\ \hline 180 \\ + 240 \\ \hline 258 \end{array}$$

Repeated addition

$$40 + 40 + 40 + 40 + 40 + 40 = 240$$

$$\begin{array}{r} 69 \\ 3+3+3+3+3+3=18 \end{array}$$

$$240 + 18 = 258$$

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$$

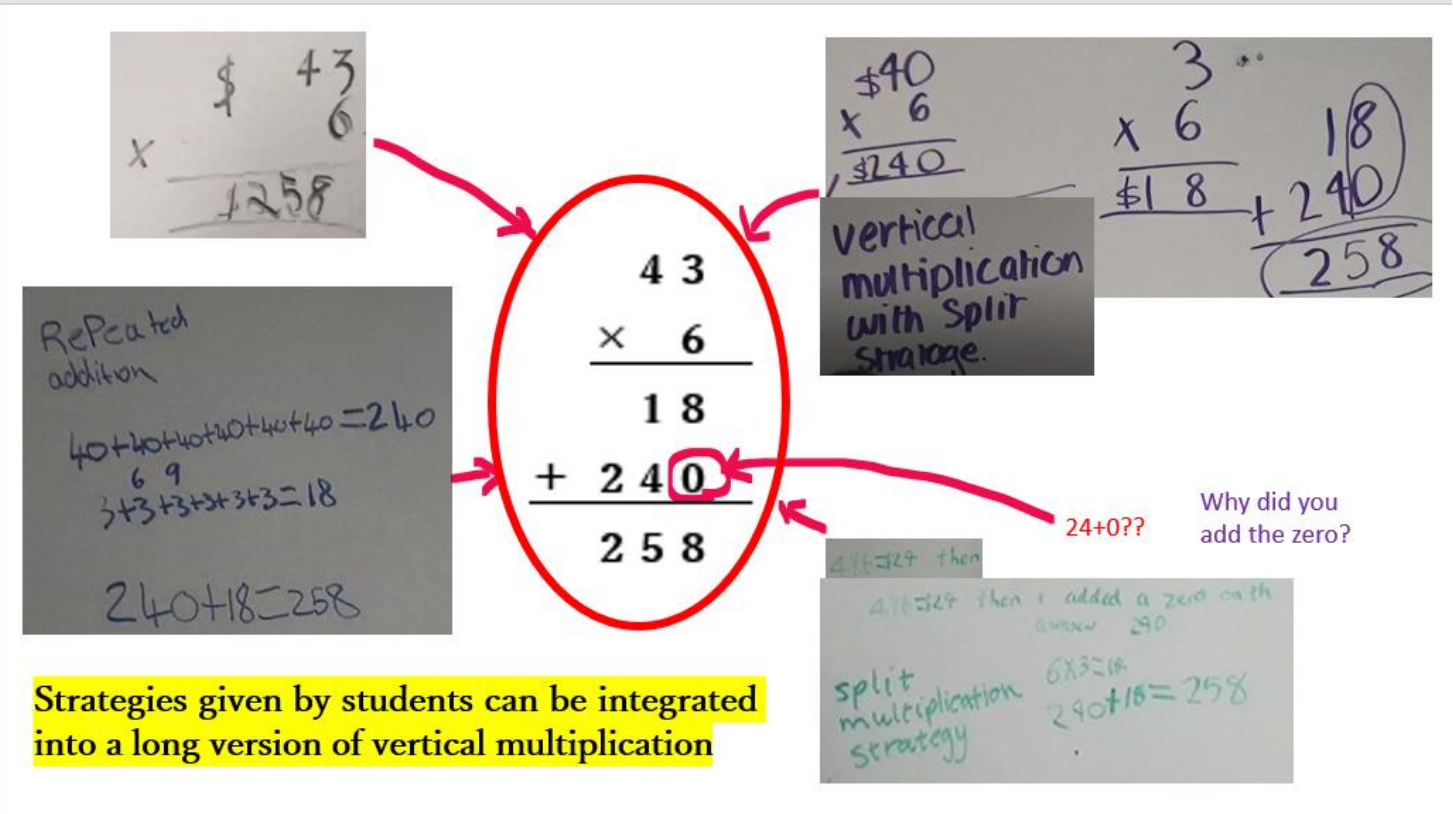
Vertical multiplication with split strategy.

24+0??
Why did you add the zero?

split multiplication strategy
 $4 \times 6 = 24$ then
 $30 \times 6 = 180$ then
 added a zero on the new 240
 $6 \times 3 = 18$
 $240 + 18 = 258$

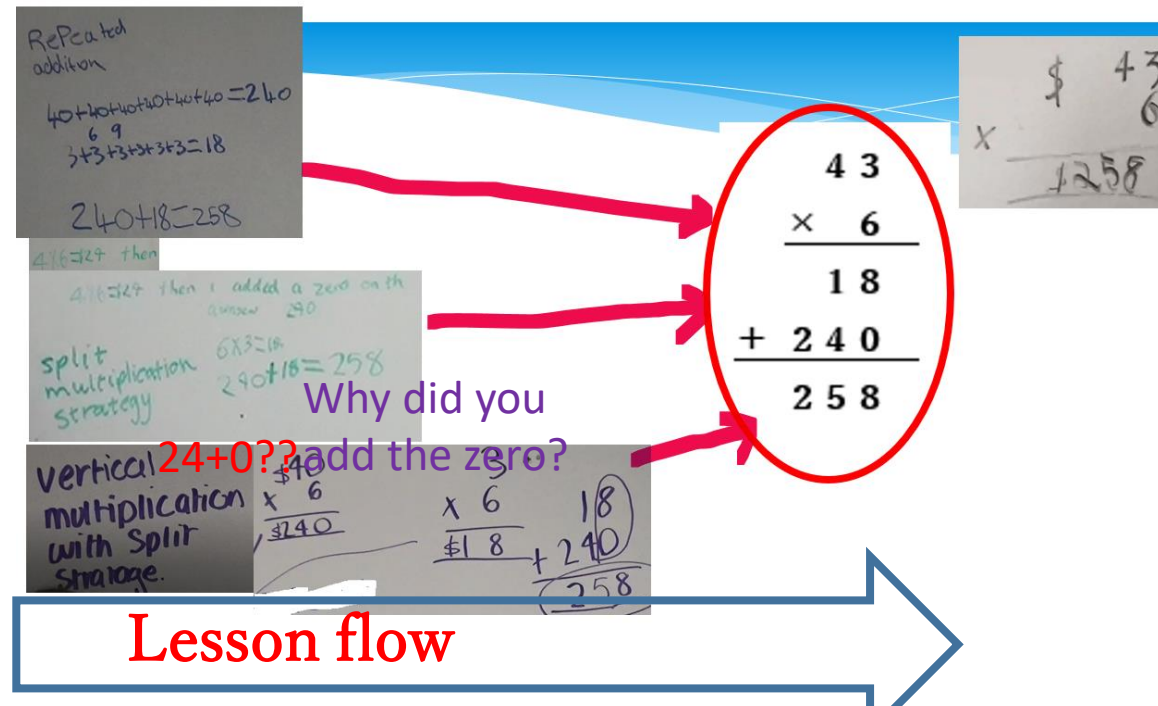
Why did you add the zero? 24+0??
 Strategies given by students can be integrated into a long version of vertical multiplication

Algorithms should not be taught without understanding. Therefore the direction of the arrow is important, it is from strategies (S1 to S5) to formal algorithm, but not the other way around.



Because, an algorithm should not be following some procedure without any understanding.

- Lesson flow is from students' strategies to the formal algorithm.
- Felicity sensei's lesson ended just before the algorithm. Never mind, it's okay.
- Because the lesson flow is appropriate, so that this is a good lesson in terms of teaching of algorithms



In other words, although the lesson could be an appropriate place to introduce the long vertical multiplication algorithm, it should be okay in the next lesson or later lessons, as far as students can recall their strategies (S1 to S5)

Repeated addition
 $40+40+40+40+40=200$
 6×9
 $3+3+3+3+3=15$
 $200+15=215$

split multiplication strategy
add a zero with
 240
 $6 \times 3 = 18$
 $240 + 18 = 258$

vertical multiplication with split strategy
 $\begin{array}{r} 40 \\ \times 6 \\ \hline 240 \end{array}$
 $\begin{array}{r} 30 \\ \times 6 \\ \hline 180 \end{array}$
 $\begin{array}{r} 240 \\ + 18 \\ \hline 258 \end{array}$

Why did you add the zero?
 $24+0??$

$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$

$\begin{array}{r} \$ 43 \\ \times 6 \\ \hline 258 \end{array}$

Lesson flow

Anyhow, why do we teach algorithms, then ?

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ + 240 \\ \hline 258 \end{array}$$

Handwritten multiplication of 43 by 6, including a dollar sign. The result is 258.

$$\begin{array}{r} \$ 43 \\ \times 6 \\ \hline \$ 258 \end{array}$$

Handwritten multiplication of 43 by 6, with a circled 6 and the text "vertical multiplication". The result is 258.

$$\begin{array}{r} \textcircled{6} \\ 43 \\ \times 6 \\ \hline \$ 258 \end{array} \quad \text{vertical multiplication}$$

Handwritten multiplication of 40 by 6, with the text "vertical multiplication with split strategy". The result is 240.

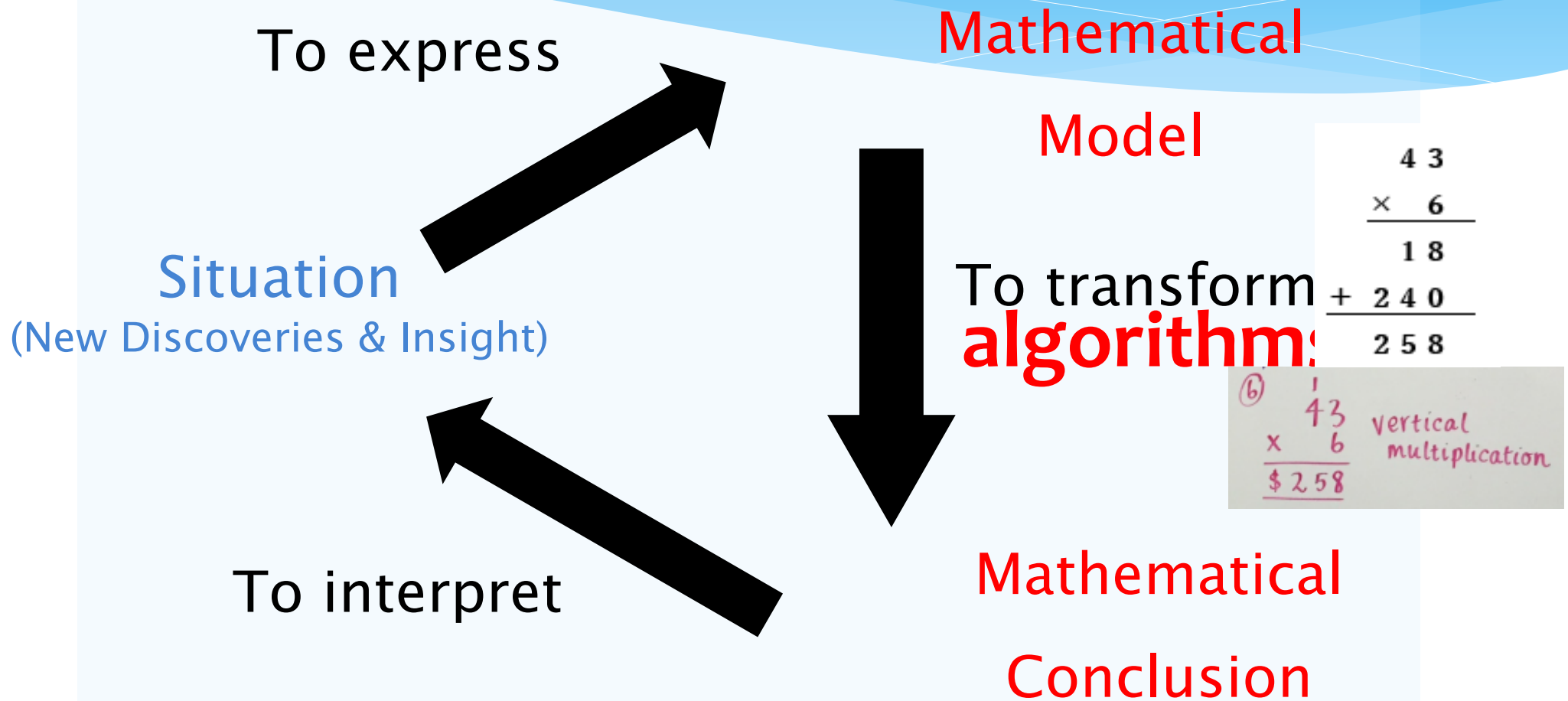
$$\begin{array}{r} \$ 40 \\ \times 6 \\ \hline \$ 240 \end{array}$$

vertical multiplication with split strategy.

Handwritten multiplication of 30 by 6, followed by addition of 180 to get 258. The 180 is circled.

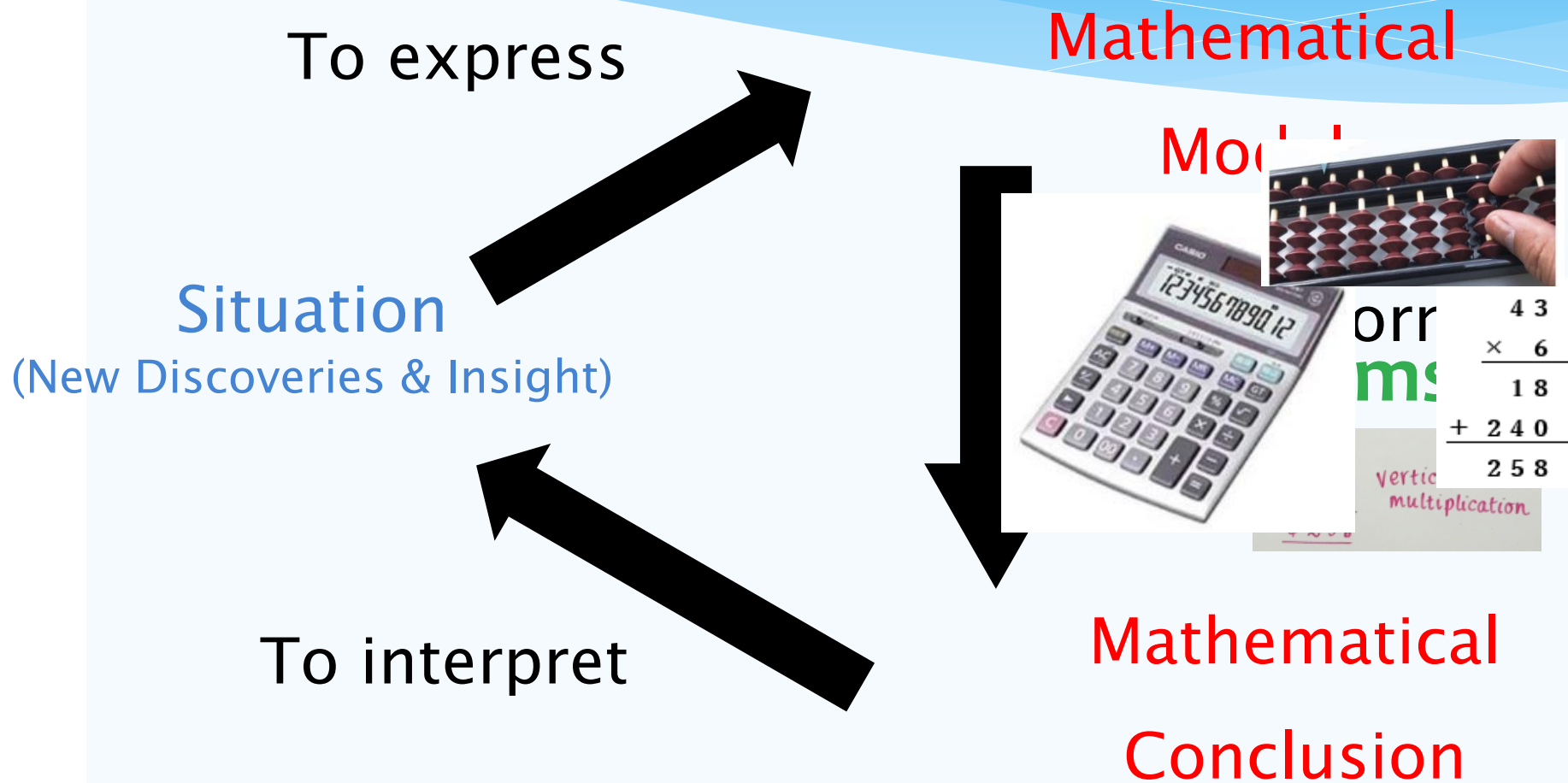
$$\begin{array}{r} 30 \\ \times 6 \\ \hline \$ 180 \end{array} + 240 = \underline{\underline{258}}$$

Algorithms are located at the transforming process



Why do we teach algorithms ?

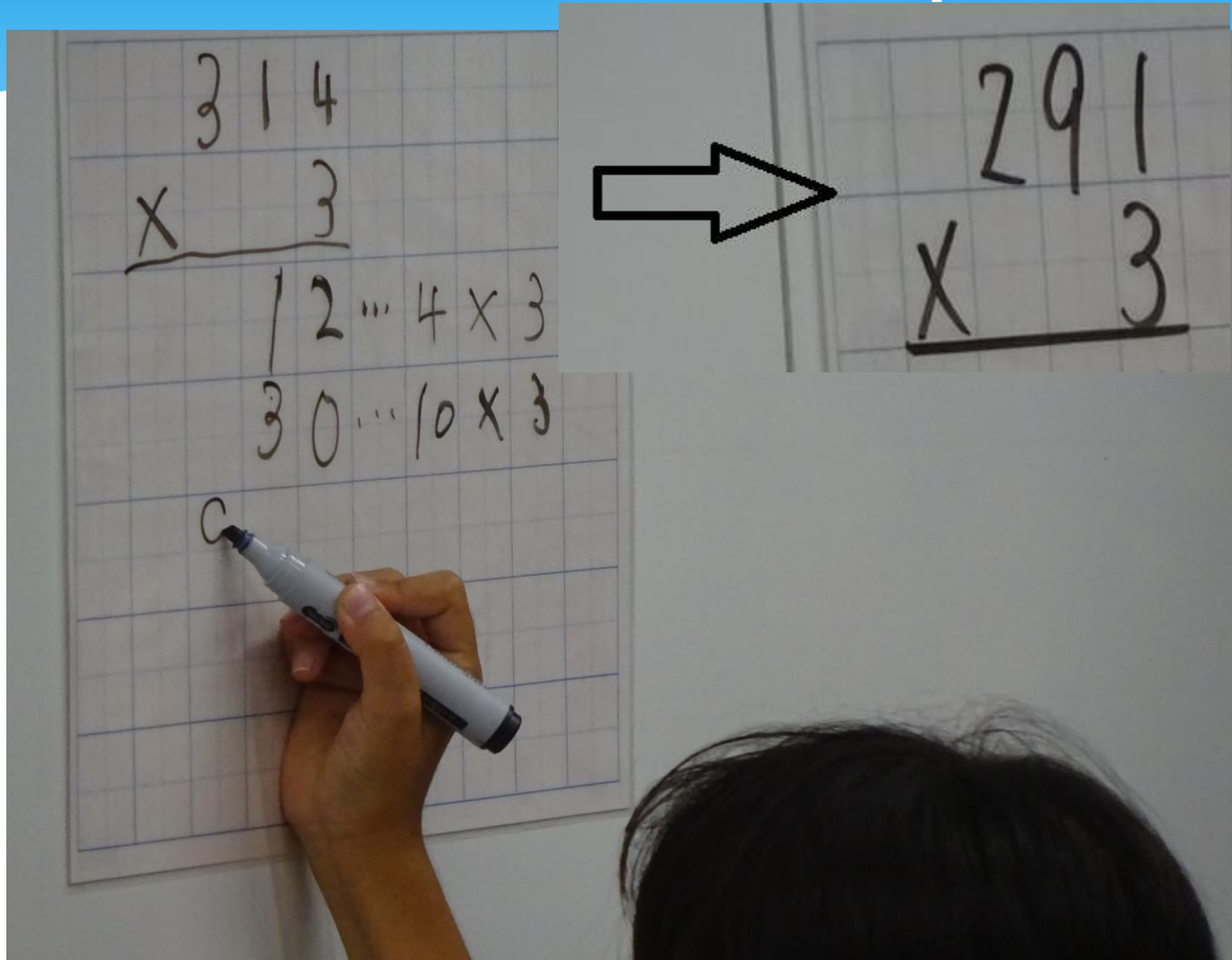
Even we could use a calculator (an abacus?)



why do we learn algorithms?

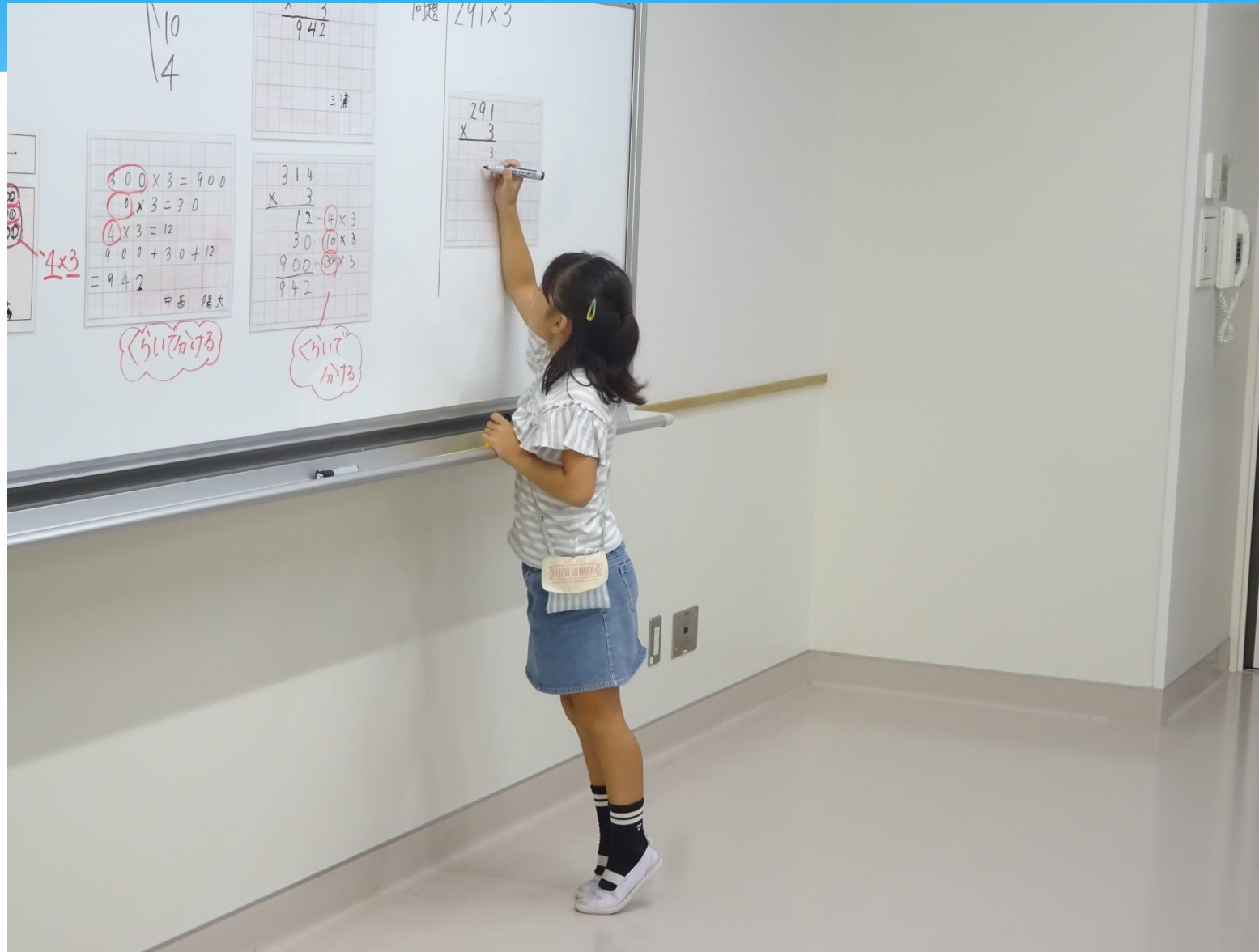
- * Calculators : we can NOT see the process of calculation
- * **Algorithms : we can see the process of calculation**
- * If we do some mistakes, we can see where it is, we can see the reason why we did mistake
- * **Algorithms allow us to diagnosis which parts are inappropriate and we could correct the mistakes by our selves**

Screen shouts from Japanese research lesson

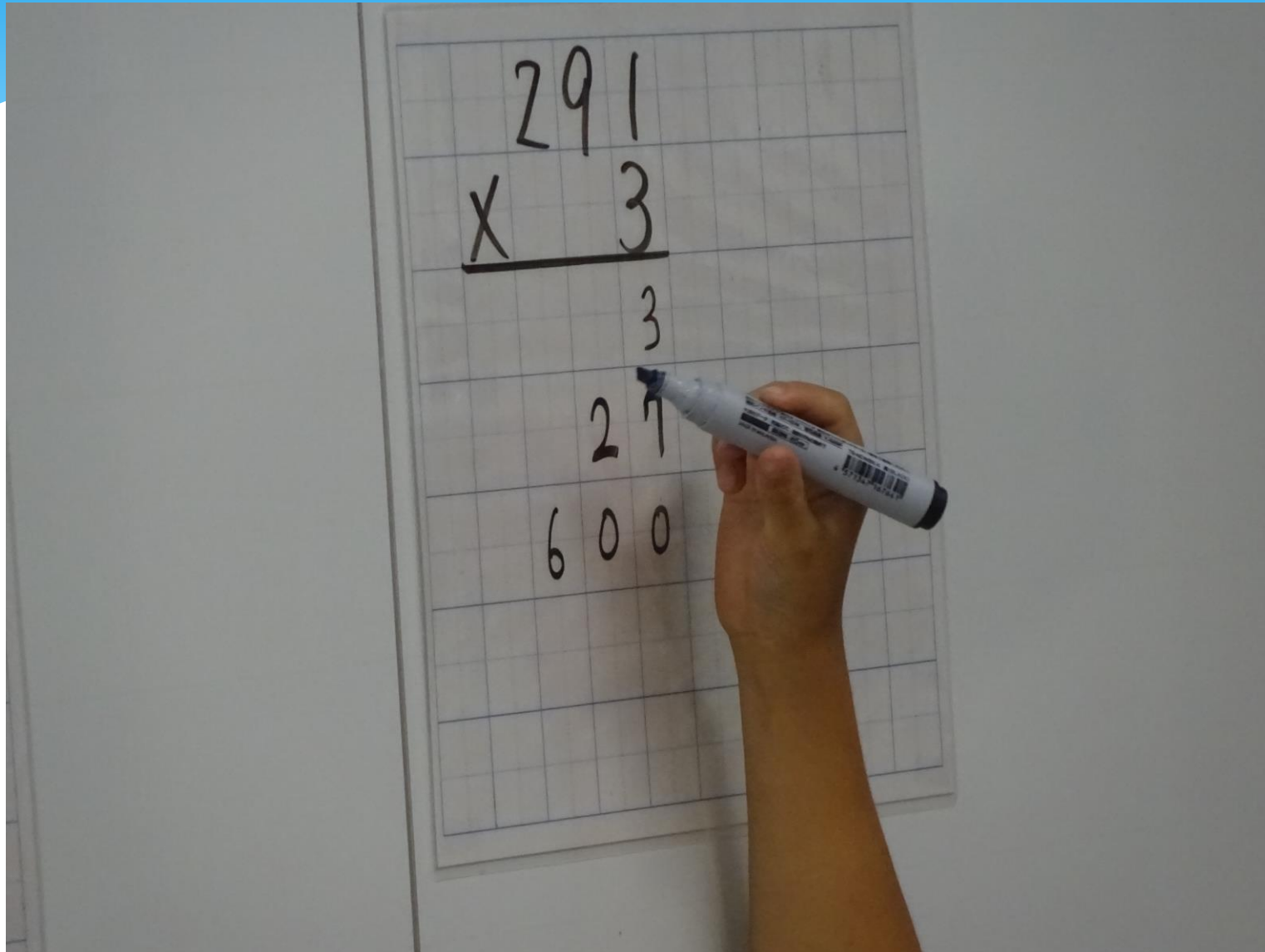


- * The task is 314×3
- * At the end of the lesson, students apply to 291×3

A girl came to the front to explain her solution



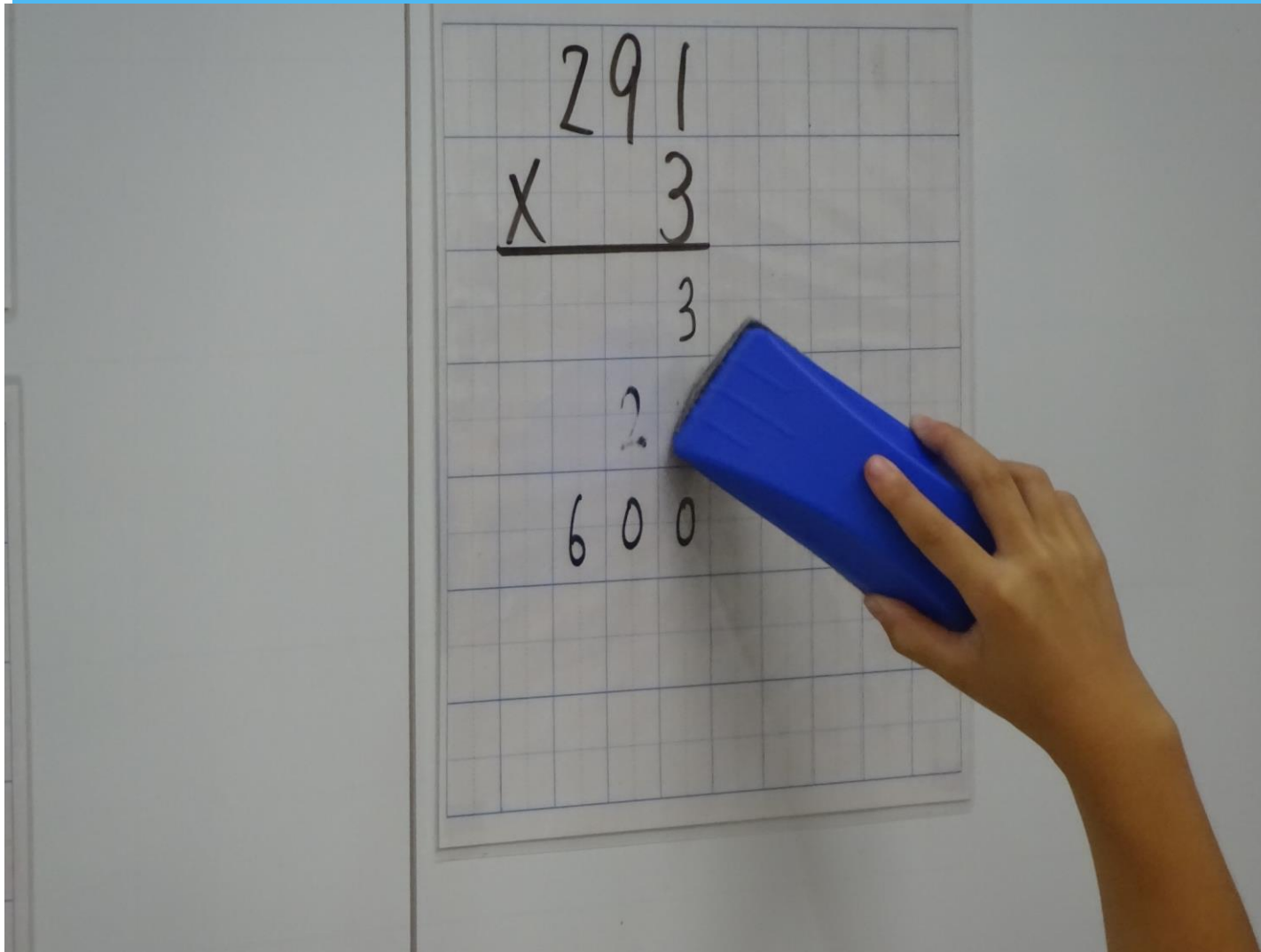
A girl came to the front to explain her solution



* She put
“27”

* Classmates
stared at
her
movements
with silence.

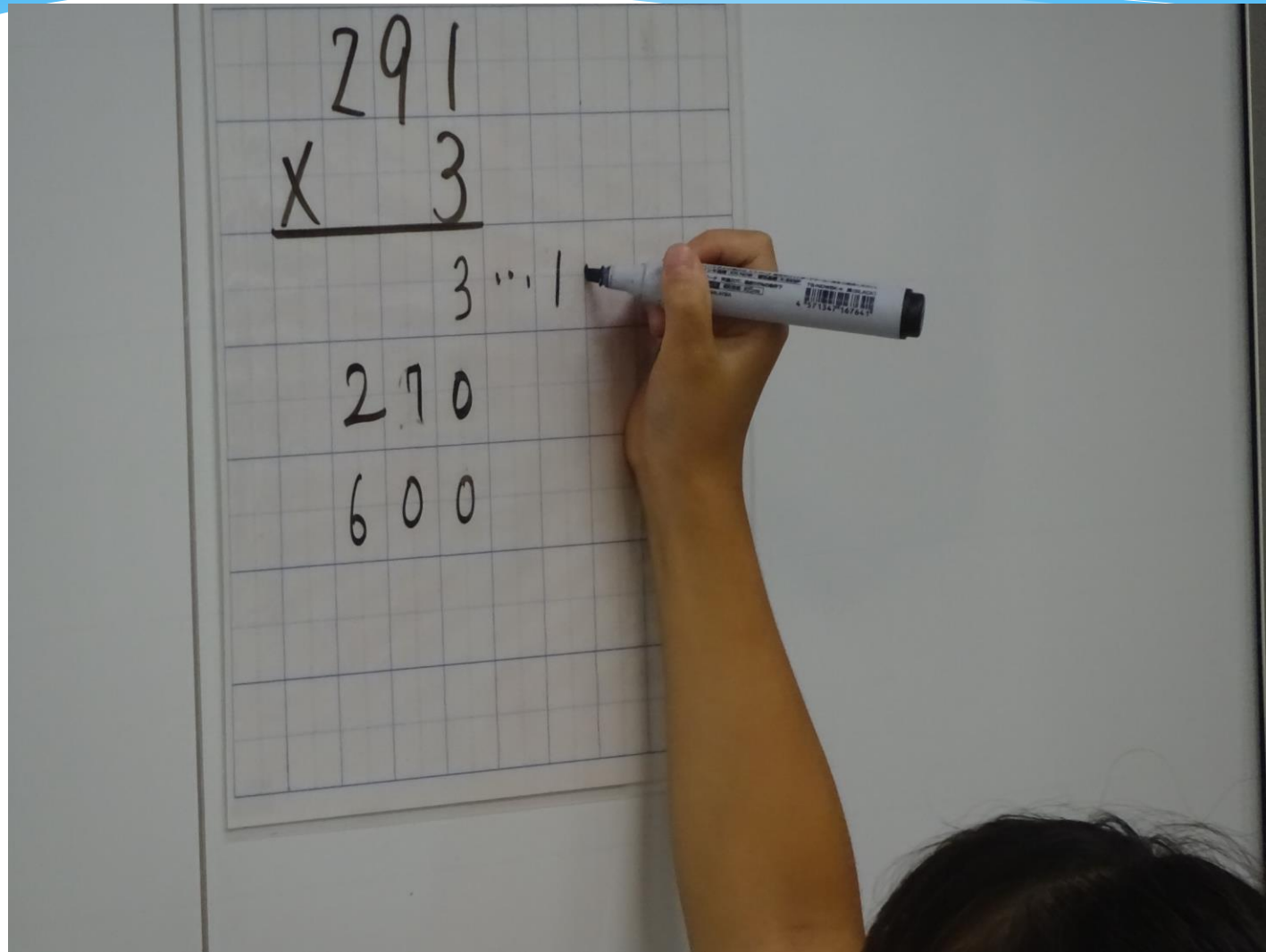
A girl came to the front to explain her solution



* She erased
“27” out

* Classmates
still stared
at her
movements
with silence.

A girl came to the front to explain her solution



* She put
“270”

* And going
to write
reasons for
numbers in
right side

A girl came to the front to explain her solution

$$\begin{array}{r} 291 \\ \times 3 \\ \hline \end{array}$$

3 ... 1x3
270 ... 9x3
600 ... 200x3

*Indeed, 270 is not 9 times something, but...

A girl came to the front to explain her solution

$$\begin{array}{r} 291 \\ \times 3 \\ \hline 3 \dots 1 \times 3 \\ 270 \dots 90 \times 3 \\ 600 \dots 200 \times 3 \\ \hline \end{array}$$

*She put
270 and
 90×3

A girl came to the front to explain her solution



* “How was my work?”
Classmates
clap their
hands.

Classmate commented on her work and attitude



- * “I was impressed by her attitude.
- * She did a great job”

Classmate commented on her work and attitude



- * “I did same mistake, too.
- * She showed us how to correct it.”

Unfortunately, the final summary did not mention about values of algorithms

	2	9	1		
X			3		
<hr/>					
		3	...	1x3	
	2	7	0	...	90x3
	6	0	0	...	200x3
<hr/>					
	8	7	3		

13-43反り

2けた \times 1けたと同じように
くらいで分けた
九九をつかった。

2けた \times 2けた
3けた \times 2けた
数を大きく

The final summary did not mention about values of algorithms

$$\begin{array}{r} 291 \\ \times 3 \\ \hline 873 \end{array}$$

3 ... 1×3
270 ... 90×3
600 ... 200×3

Reflection

We use the number facts from 1×1 to 9×9

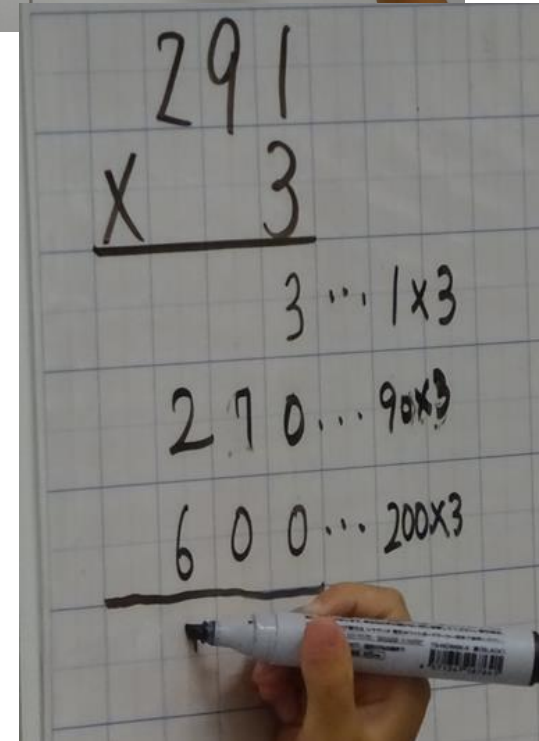
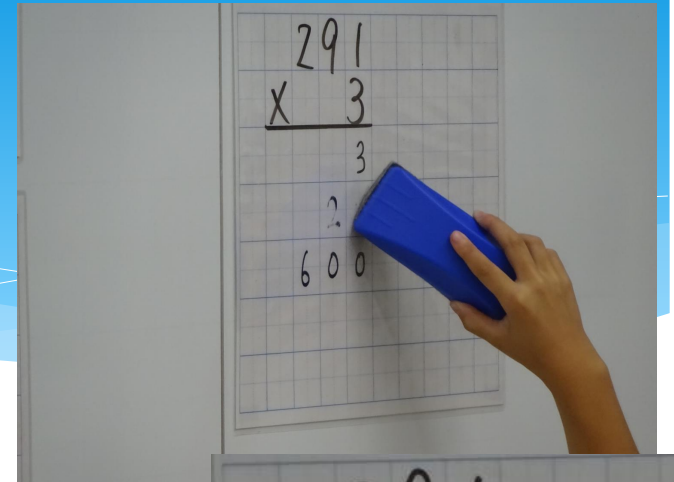
We splitted numbers again as we did for 2dists times 1 digit

I want to try "2 digits \times 2 digits" "3 digits \times 2 digits" "larger numbers to try"

So I talked about the value of algorithms as a final commentator

The Big Idea

- * Values of algorithms:
- * We can see the process
- * We can identify which part is inappropriate or incorrect, so that we can correct them by ourselves
- * Algorithms work for
- * Self-diagnosis
- * Self-treatment



Value of algorithms

- * This lesson (or next lesson) is an appropriate place for teacher to introduce the longer version of vertical multilocation.
- * Introducing the longer version of vertical multilocation, Neriage should be richer.
- * **Through teaching of algorithms,**
We could teach students how to become a person who can do Self-diagnosis and Self-treatment

Finally,

Why we do Structured problem-solving lesson ?

- * Because we want to teach mathematics through problem solving.
- * So why “problem solving”?
- * Because we want to educate students to become independent problem solvers or independent thinkers.

Thank you for Felicity sensei + planning team. And for your students:
You are genuinely independent thinkers!

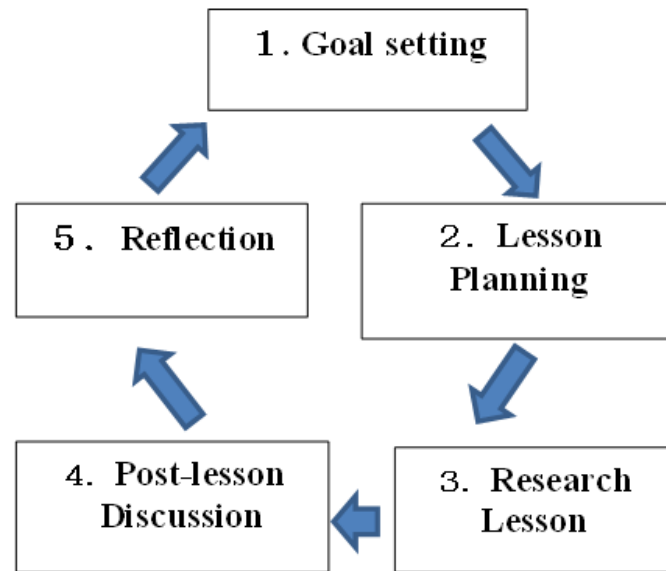


To prepare my comments, I dug up deeply the video-taken research lesson and post-lesson discussion, lesson plan, student notes and Australian curriculum. Through this experience, I become aware of that:

Webinar Lesson Study has new features, or a new future!

We have never experienced it before.

Ordinary Lesson Study



Webinar Lesson Study



It seems to be integrated or combined