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Fibonacci: A structured investigation

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It is hard to imagine that, eight hundred years on, the study of Fibonacci could affect the lives of teenagers in Australia. Or is it? A mathematics class of more able Year 9 students in a regional city of Western Australia feels that it has happened to them. Thirty-two students submitted a Fibonacci task (See Appendix 1) as a mathematics assessment, with many of them acknowledging that they now view the world differently as a direct result of this work.

The task

The task was divided into four parts, each with well-scaffolded instructions. Part 1 set the scene, with students researching Fibonacci, using a variety of resources. In Part 2, students were asked to find connections between the Fibonacci numbers and the patterns breeding of rabbits. Part 3 introduced technology by using a spreadsheet to generate the Fibonacci numbers and their ratios to reach the Golden Ratio. In Part 4, students explored the appearance of the Golden Ratio in a number of human endeavours. They were also asked to reflect on their learning.

Part 1: Biographical paragraph

The Fibonacci task begins with a paragraph of biographical details. Students are expected to produce a Word document, so many choose to write notes straight onto the computer. The templates provided by our school library staff help students to produce appropriate references, which for many is the first time in their high school career this has been required. Students are encouraged to find and use an encyclopaedia, as well as using the internet.

Over the years, our library staff has compiled a large collection of book resources and internet sites appropriate to this task. Imagine our delight when we found The rabbit problem by Emily Gravett (2009) this year, with its gorgeous illustrations, quirky humour and even a knitting pattern!

We set up a structure for our notes, consult a number of references, and then draft a paragraph. Parents are often co-opted to edit the work, and we get some interesting comments. There is also appreciation from other areas in the school as we develop some of the research and editing skills not normally addressed in mathematics. In addition, many students realise that the Hindu Arabic number system that we use has not always existed.

Part 2: Mathematical modelling

In class, we began by discussing how mathematics can model a real situation, for example, rabbit breeding. A model must also take into account any underlying assumptions. Unlike our model, in the real-world rabbits will not be born in male/female pairs, nor live forever, have enough food and escape predators. However, we can learn a great deal by looking at the patterns that emerge.

To follow the model accurately and communicate clearly, we decided to use a visual code for the various parts of the rabbit life-cycle (see Figure 1). In the first month, we drew a very small face with big ears to represent the first pair of baby rabbits. Next month, the baby rabbits turn into teenage rabbits with bigger faces (and lots of pimples!). By the third month the teenage rabbits are adults, and are represented with larger faces and a bow-tie. This visual code made it very easy to keep track of the various generations.

The first pair of rabbits was allocated a colour, and their life cycle over each month was recorded down the first column. As they reached breeding age, their progeny appeared in the next column, with a different colour. Every month, each adult pair produced new babies, the babies became teenagers, then adults...and life continues.

By drawing pictures and using colours, the patterns start to emerge. Students begin to see what is happening and can explain the process (even though they soon run out of space).

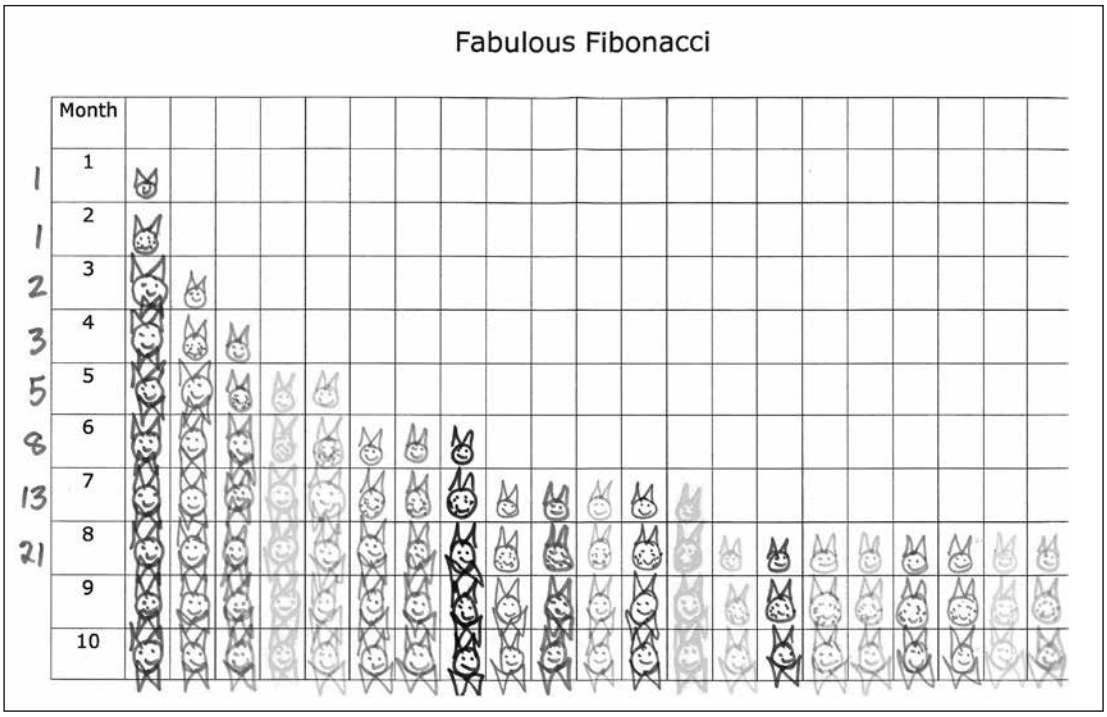

Figure 1. Fibonacci rabbits.

Students then have the opportunity to use their diagrammatic representation to formalise their thinking, using an academic style of writing. Understanding certainly seems to come more easily through the visual representations that students have created for themselves, and with their own personal thinking about the patterns already in place. We see a synthesis of ideas from drawings and references; the research is taken seriously due to the investment already made in the drawings.

Part 3: Spreadsheets

Having set up a simple spreadsheet to calculate the Fibonacci numbers, students then calculate the ratio of one Fibonacci number to the previous one, thus revealing the Golden Ratio.

The spreadsheet is a tool that increases the efficiency of the investigation. Since spreadsheets feature prominently in many workplaces, taking the opportunity to reinforce skills such as formatting, rounding, creating and copying formulas is a sensible choice.

Part 4: Research on the Golden Ratio

Students were asked to seek evidence of the Golden Ratio in architecture, art, music and nature. It is in this part of the task that students see mathematics in ways that they have never thought of before. They follow their own interests, with the musicians spending more time on the ratio's appearance in music, while the outdoor types focus more on nature. For many, the discovery that mathematics is not limited to classroom activities is a huge surprise.

Reflection

When asked to evaluate their own performance on the task, students were honest. The half dozen that did not make much of an effort said so. Others recognised that their personal organisation could be improved. Most were able to appreciate the effort that they had applied to the task and feel satisfied with the results. Then there were those whose view of mathematics changed.

> As I was answering each section of the assignment I came across new and interesting bits and pieces of information every time and was astounded at how fascinating the different facts and reasoning were – before I really was dreading it. I believe I've learnt a gigantic amount of information and I am finding myself looking at certain things differently: looking for the golden ratio, or some similarity between a particular formation and the Fibonacci sequence. With my artwork, I think I'm going to try and experiment with the golden rectangle/rectangle/proportions. When I draw trees, I'll remind myself of the rabbit sequence and the growth points of that tree that grow in the same sequence. When I draw faces I'll think of Mona Lisa and her 'perfect' proportions. Overall though, the Fibonacci assignment has intrigued me and I believe I've come out the other side of it with a deeper understanding of the world we live in, both man -made and natural."

> "I have worked to the absolute best of my ability in this task. I don't feel as if I've had any problems completing the task, but I did learn some new valuable techniques. In part three where we had to compose the spreadsheet, I learnt about duplications and how to do equations, which is very useful. Doing the whole task has made me feel a sense of accomplishment and pride. This is definitely something I'd feel proud to display in my certificate folder.

As a teacher, I can imagine no better feeling than watching a group of students who have significantly developed research and spreadsheet skills, and who have been immersed in a mathematical modelling situation. The fact that the students took pride in their efforts and achievements is important. Significant amounts of time and effort were spent using language to discuss mathematical ideas. Now these students see the world and mathematics in a completely different way.

References

Devlin, K. 2012 *The Man of Numbers: Fibonacci's Arithmetic Revolution,* Bloomsbury, Great Britain.

Gravett, E. (2009). *The Rabbit Problem.* New York, US: Simon & Schuster Children's Publishing.

Appendix 1

Fibonacci task

Part 1

You are to conduct some research about Fibonacci. Your research should answer the following questions:

- Who was he?
- When did he live?
- Where did he live?
- What did he do?
- Why is he famous?

A Using the correct form of referencing, (see Connect, diary, library), construct a page of notes on this topic from:
  a) A book
  b) An encyclopaedia
  c) A web page

B Using the three sets of notes, write a clear, concise response to the five questions, giving the three references at the end of the response. Mark this page DRAFT, then print it.

C Ask someone (friend, parent, teacher) to edit the response for you, ensuring that the person signs the record page, as well as making notes on the draft.

D Using feedback, compose the final response.

Part 2

The Fibonacci numbers are the numbers generated in a sequence when you start with 1, 1, then add the two previous numbers to get the next in the sequence e.g., 1, 1, 1+1=2, 1+2=3, 2+3=5,...

E List the first twenty Fibonacci numbers in order.

F Using two difference sources, as well as your class activity, construct notes, with references, on the connections between the Fibonacci numbers and rabbits.

G Write your own description and explanation of the Fibonacci numbers using the rabbit situation. You may wish to include a table or a diagram. Include both references you used.

Part 3

H Generate a spread sheet called Fibonacci numbers.

- Start in cell $A3$ with 1, cell $A4$ with 1.
- Use a formula in cell $A5$ to generate the next one (= $A4+A3$).
- Copy the formula down the page to generate the first 50 Fibonacci numbers

We are going to consider how the numbers in the sequence compare to one another by looking at the ratio of a number to the previous one, for example $\frac{2}{1}$ or $\frac{3}{2}$.

I In cell $C2$, put the heading Ratio.
  In cell $C4$, use a formula to find the ratio (= $\frac{A4}{A3}$).

Set the format of the cell so that the ratio is given to four decimal places. Copy this formula down the sheet for the rest of the numbers.

J Ensure your name is part of the file name, submit this spread sheet onto Connect.

K Make some comments on what you see.

Part 4

L Using three different sources, not all of which are internet based, make notes, including references, on the Golden Ratio. You need to include examples of the ratio from the fields of architecture, art, music and nature.

M For each of the four fields of architecture, art, music and nature, explain an application of the Golden ratio using diagrams where appropriate and citing all references.

N Reflect on your performance in this task—was it the best you could do? How could you have improved? Were there any specific problems? What bits did you find hard to do? Did you learn anything?

Item	Possible	Score
A Book – reference	1	
Notes	3	
Encyclopaedia – reference	1	
Notes	3	
Web– reference	1	
Notes	3	
C Editor – name	1	
Signature	1	
D Response – references	1	
Questions	5	
E Fibonacci numbers listed	1	
F Rabbits – reference	1	
Notes	3	
Second – reference	1	
Notes	3	
G Rabbit explanation – references	1	
Explanation	5	

Item	Possible	Score
I Spreadsheet – titles	1	
Numbers	3	
Ratios	3	
K Comments	3	
L Golden ratio – reference	1	
Notes	3	
Reference	1	
Notes	3	
Reference	1	
Notes	3	
M References	1	
Architecture	3	
Art	3	
Music	3	
Nature	3	
N Reflection	5	
Total	75	

Comment	