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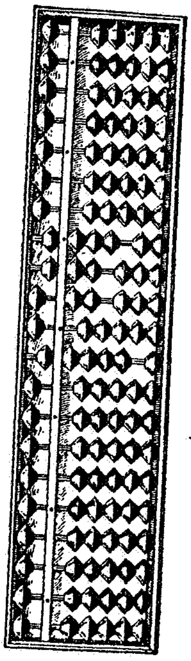
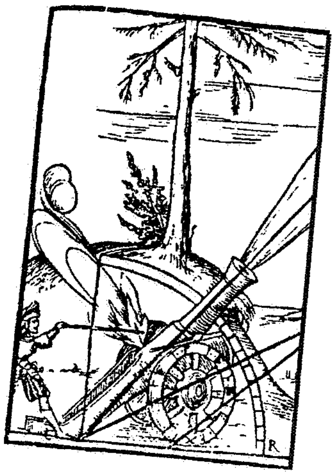
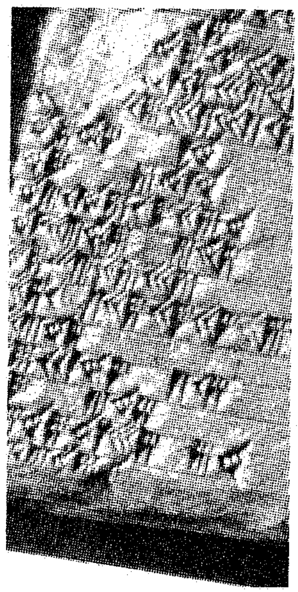




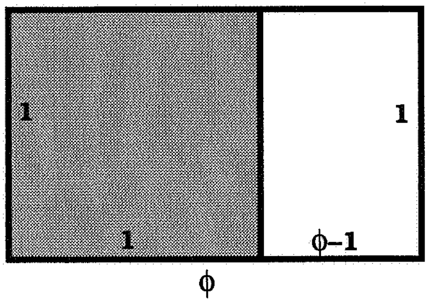



PITAGORAS

THE GREEKS GO FOR GOLD!

A special rectangle

Suppose we take a rectangle R, and inscribe a square at one end. This will leave another rectangle R'. Now suppose that R and R' have the same shape. What are the dimensions of R?



A closer look ...

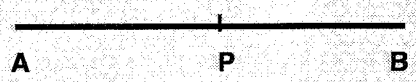
Since we are only concerned with the shape, we may take R to have length ϕ and width 1. Then R' will have length 1 (vertically) and width 1 (horizontally). Since R and R' have the same shape, $\phi/1 = 1/(\phi-1)$.

This simplifies to $\phi(\phi - 1) = 1$, or $\phi^2 - \phi - 1 = 0$.

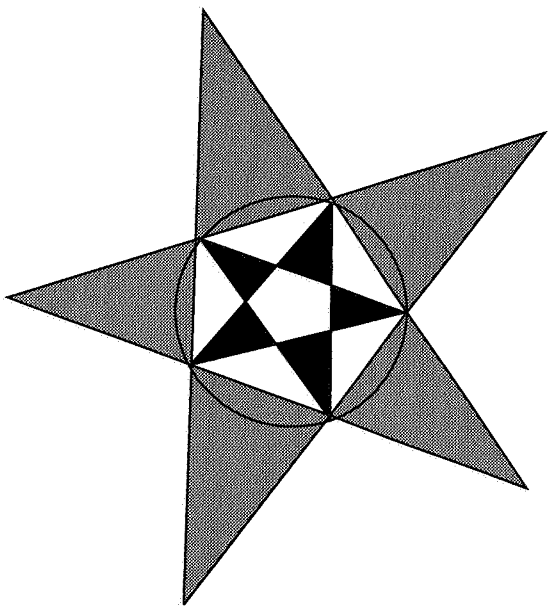
Using the quadratic formula, or a calculator, we get

$$\phi = (1 + \sqrt{5})/2 \approx 1.618\ldots .$$

The special number ϕ (= $\phi/1$) is called the *golden ratio*. The Greek letter ϕ (phi) is used in memory of Phidias, an ancient Greek sculptor, who used this ratio in idealizing human proportions. The above rectangle is called the *golden rectangle*.



1. In Euclid's *Elements*, Book VI, Proposition 30 we find: If point P divides the segment AB internally in the ratio $AP: PB = AB: AP$, then P is said to divide AB in the *golden ratio*. Show how the number ϕ occurs here.



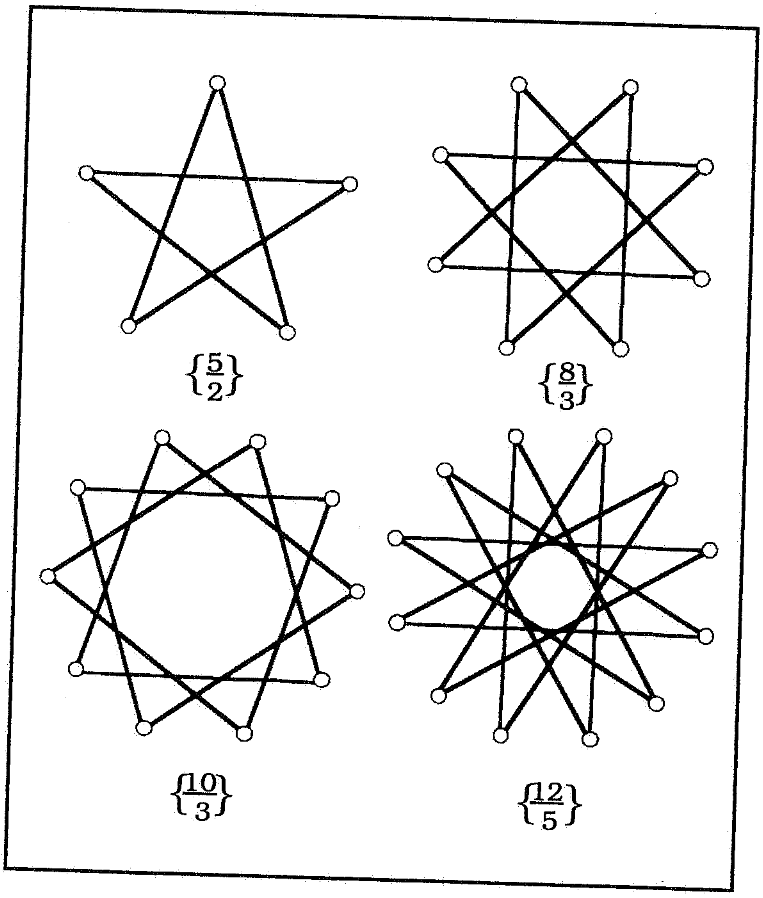
The number ϕ

The number ϕ has had a long and honourable history. The Pythagoreans not only discovered it but realized that it is irrational (or incommensurable) and embodied it in the star pentagon or *pentagram*, which became their secret sign. After Euclid studied its construction, the golden ratio became the object of study and application by mathematicians, philosophers, artists and architects throughout the centuries to the present day, not only for its own sake but also for its association with the ideal of beauty. One mathematician, Pacioli, referred to ϕ as 'the divine proportion'. Some of this history is graphically documented in the Walt Disney film *Donald in Mathemagic Land*. The mathematics of ϕ continues to fascinate those who take the time to investigate it.

YOU'RE A STAR POLYGON!

Pathway to the stars

Imagine n basketballers standing equally spaced around a circle. A ball is passed from a given basketballer to every ath player in the circle until the ball returns to the original basketballer. The path traced out is a *star polygon*, or regular *n-gram*. We may assume that n and a are relatively prime (that is, they have no common factors other than 1), and that $n > 2$. These conditions will become obvious by substituting some simple values for a and n. Such a star polygon is symbolized by $\{\frac{n}{a}\}$. The most famous star polygon is $\{\frac{5}{2}\}$, known to the Pythagoreans as the *pentagram*. When $a = 1$, we have a regular polygon $\{n\}$; we can think of this as a special star polygon $\{\frac{n}{1}\}$.



1. Choosing some values of n and a, draw some star polygons of your own. Can you show that the star polygon $\{\frac{n}{a}\}$ is identical to the star polygon $\{\frac{n}{n-a}\}$? Show that if n is prime, there are $\frac{1}{2}(n-1)$ regular n-grams. What happens if n is not prime?

2. What meaning do you attach to the number n for the polygon $\{n\}$? Can you attach meanings to the numbers n and a for the polygon $\{\frac{n}{a}\}$?