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Author(s)	Ann Heirdsfield
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Mental Methods Moving Along

How can we encourage students to take ownership of their own mental strategies? **ANN HEIRDSFIELD** explores this issue in relation to students making sense of numbers using their own natural skills.

> 'How many days are there in the year?'
> 'Three hundred and sixty-five,' said Alice.
> 'And how many birthdays have you?'
> 'One.'
> 'And if you take one from three hundred and sixty-five, what remains?'
> 'Three hundred and sixty-four, of course.'
> Humpty Dumpty looked doubtful.
> 'I'd rather see that on paper,' he said.
> (Lewis Carroll, *Through the Looking Glass*)

Hopefully, this absurd scenario would not be witnessed in an Australian classroom. Have we seen examples of the type, '3000 – 2977' given to children to solve using the traditional pen and paper method? When my son was in Year 4, he was asked to solve such an example as a challenge exercise (he had just learnt the written algorithm for four-digit subtraction involving regrouping). He solved the example mentally (as he had no idea how to use the algorithm), and then inserted the zeros, cross outs, and so forth in the appropriate places to record his response. Hopefully, we are not condemning our children to be totally reliant on traditional pen and paper methods. In the National Statement (Australian Education Council, 1991), the need for alternative written computation strategies was recognised.

> People with good number skills… use a variety of informal but reliable written methods which are appropriate to the person, to the numbers involved...and to the context… Therefore, it is inappropriate to focus upon computation in isolation from the context of its use (p. 109).

Around the world and within Australia there is growing support for postponing or even abolishing the teaching of standard written algorithms. Instead, there is a growing tendency to focus on number sense and the development of mental computation as a major component of computation. By mental computation I do not mean basic facts recall, or calculation of extended number facts (e.g. 30 + 20 = 50, because 3 + 2 = 5), although these may be involved; mental computation refers to multidigit calculations performed mentally. In The Netherlands, where there is no equivalent word for 'mental', the phrases 'working in the head' and 'working with the head' are used. The focus, in this article, is 'working with the head'.

Why is mental computation important?

There is increasing awareness of the importance of the role of mental computation in the numeracy component of mathematics curricula. Many researchers (e.g. Cobb & Merkel, 1989; McIntosh, 1990; Reys & Barger, 1991; Sowder, 1990) argued that mental computation is a valid computational method in which children can learn how numbers work, make decisions about procedures, create strategies and explore rich fields of mathematical experience. Moreover, Kamii, Lewis, and Jones (1991) recommended that children should be free to formulate their own mental strategies as they understand procedures better if they are allowed to construct procedures in line with their own natural ways of thinking. Further, they become active participants in their learning. By being involved in developing their own strategies, children manipulate quantities, rather than symbols, they make decisions about the procedures they use, and they develop number sense.

It has been suggested that eighty percent of computation used in everyday life is done mentally (Clarke & Kelly, 1989). After all, some calculations are most easily solved using mental strategies, rather than traditional written procedures (e.g. the exercise solved by my son).

The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) emphasised the importance of mental computation in the curriculum.

> People who are competent in mental computation tend to use a range of personal methods which are adopted to suit the particular numbers and situation. Therefore, students should be encouraged to develop personal mental computation strategies, to experiment with and compare strategies used by others, and to choose from amongst their available strategies to suit their own strengths and the particular context (p. 109).

The value of mental computation is to promote understanding and flexibility in number and operations, that is, to develop number sense. It has been argued that children (and adults) who possess number sense use a variety of strategies for different number combinations, and that number sense improves when children are involved in mental computation activities. Note that I do not suggest teaching mental computation strategies. Other researchers (Kamii, Lewis, & Jones, 1991; Reys, Reys, Nohda, & Emori, 1995; Thompson, 1999) have recommended that children should not be taught formal computation methods. McIntosh (1996) suggested that teaching mental computation strategies would be just as futile as has been the teaching of pen and paper procedures. Rather, children should be given opportunities to develop their own strategies. What mental computation strategies do children use?

Mental computation strategies

A variety of research projects have identified strategies children use to solve mental addition and subtraction problems (e.g. Beishuizen, 1993; Cooper, Heirdsfield & Irons, 1996; Reys, Reys, Nohda & Emori, 1995; Thompson & Smith, 1999). I present these strategies (Table 1), not so that teachers can check that their students are employing them; but awareness of these strategies will help understand children's explanations. Listening to children's explanations is an important part of teaching, and is a focus of several research projects, for instance, *Count Me In Too* and the *Early Numeracy Research Project*. No categorisation scheme, however, can encompass the variety of strategies that all children use.

Table 1. Mental addition and subtraction strategies

Strategy		Example	
Separation	right to left	28 + 35:	8 + 5 = 13, 20 + 30 = 50, 63
		52 – 24:	12 – 4 = 8, 40 – 20 = 20, 28 (subtractive)
			4 + 8 = 12, 20 + 20 = 40, 28 (additive)
	left to right	28 + 35:	20 + 30 = 50, 8 + 5 = 13, 63
		52 – 24:	40 – 20 = 20, 12 – 4 = 8, 28 (subtractive)
			20 + 20 = 40, 4 + 8=12, 28 (additive)
	cumulative sum	28 + 35:	20 + 30 = 50, 50 + 8 = 58, 58 + 5 = 63
	or difference	52 – 24:	50 – 20 = 30, 30 + 2 = 32, 32 – 4 = 28
Aggregation	right to left	28 + 35:	28 + 5 = 33, 33 + 30 = 63
		52 – 24:	52 – 4 = 48, 48 – 20 = 28 (subtractive)
			24 + 8 = 32, 32 + 20 = 52, 28 (additive)
	left to right	28 + 35:	28 + 30 = 58, 58 + 5 = 63
		52 – 24:	52 – 20 = 32, 32 – 4 = 28 (subtractive)
			24 + 20 = 44, 44 + 8 = 52, 28 (additive)
Wholistic	compensation	28 + 35:	30 + 35 = 65, 65 – 2 = 63
		52 – 24:	52 – 30 = 22, 22 + 6 = 28 (subtractive)
			24 + 26 = 50, 50 + 2 = 52, 26 + 2 = 28 (additive)
	levelling	28 + 35:	30 + 33 = 63
		52 – 24:	58 – 30 = 28 (subtractive)
			22 + 28 = 50, 28 (additive)

Separation strategies are ones where both numbers are split into place values, and then calculation proceeds either left to right or right to left. Aggregation refers to holding one number as a whole, splitting the other one into place values, then adding or subtracting in multiples of 10. Finally, wholistic strategies are ones where both numbers are kept as a whole. Note also that subtraction examples can be solved by an additive (build up) strategy or a subtractive (build down) strategy.

In the literature, it has been argued that aggregation and holistic strategies are more advanced than those that require the numbers to be split into place values; as there are fewer steps involved in the more advanced strategies, therefore less load on working memory. In The Netherlands, where children are taught mental procedures before written ones, they are expected to employ the aggregation strategy. However, only the better mental computers employ this strategy, possibly because it requires a high level of conceptual understanding.

Recently, I interviewed a group of Year 3 children, in relation to mental computation and other associated factors (Heirdsfield, 2001). A comparison between two children is relevant, here. Both children could solve quite complex (up to three-digit addition and subtraction) examples mentally and with a high degree of accuracy. One child, Clare used a variety of strategies (separation, aggregation, and holistic), and the other child, Mandy used the mental image of the pen and paper algorithm. Clare appeared to have well-connected knowledge of the principles of number, numeration, and operations, and number facts and computational estimation. Clare not only used a variety of strategies in mental computation, but was also able to formulate other efficient strategies when asked to do so. She viewed mathematics as something that should make sense. She was confident in her ability to solve tasks with her own strategies. In a number facts test, she was able to recall most number facts, and those she could not recall were solved by using derived facts strategies (not counting). Further, she viewed number in a variety of ways. Finally, she exhibited metacognitive strategies (e.g., evaluating and reflecting) and mastery achievement goals.

None of this was evident in Mandy. Mandy appeared to rely on teacher-taught procedures. When asked to think of alternative methods, she was able to do so only after a great deal of scaffolding, and even then preferred teacher-taught procedures. Mandy's aim for number facts was speedy and accurate recall. Although she was able to recall many facts, those she could not were solved by counting, a less efficient strategy than those used by Clare. Mandy had difficulty viewing numbers in more than one way. There was evidence that to Mandy, mathematics need not make sense. She exhibited no metacognitive strategies, and she exhibited performance (c.f., mastery) achievement goals. Thus, the distinction between the two mental computers amounted to differences in flexibility, access to connected knowledge and strategies, metacognition and affects.

Encouraging mental computation in the classroom

As has already been mentioned, it is recommended that children should not be taught formal computational methods. Rather, they should be encouraged to formulate their own. In this final discussion, some suggestions for developing children's informal strategies are described.

First, real world problems are more likely to elicit mental strategies than traditional symbolic exercises. A survey of children's interests may help identify some appropriate contexts in which to formulate word problems.

Children should be encouraged to formulate their own mental strategies and discuss and share them. Children need to be given time to experiment with a variety of procedures. All children's responses should be valued and individual responses accepted. In other words, an environment that promotes freedom to choose, experiment and share needs to be established.

Children can be encouraged to document their mental procedures, as these strategies may be efficient for written algorithms as well. In fact, children could be encouraged also to invent their own written algorithms. In particular, avoid examples of the type, 102 – 97, to be solved using the traditional pen and paper algorithm. This is far more efficiently solved using other methods. It would be interesting to ask students what method they might use — pen and paper, mental computation, or calculator.

Although number facts, place value and other understandings are important aspects for mental computation, there is some evidence that teaching them in isolation and before computation serves no purpose (e.g., Usnick & Engelhardt, 1988). These understandings seem to develop when children are encouraged to formulate their own mental strategies.

There have been some attempts to provide children with mental models for mental strategies (Beishuizen, 1993). The open number line (see Figure 1) and 99 chart (see Figure 2) may be useful to help children develop aggregation and holistic strategies. Children should be encouraged to formulate their own methods while using these models. I shall use the solution strategies for the example 65 – 39 to demonstrate the use of the open number line and 99 chart. In their simplest form, the strategies are little more than counting strategies, but they can be built on to develop efficient methods.

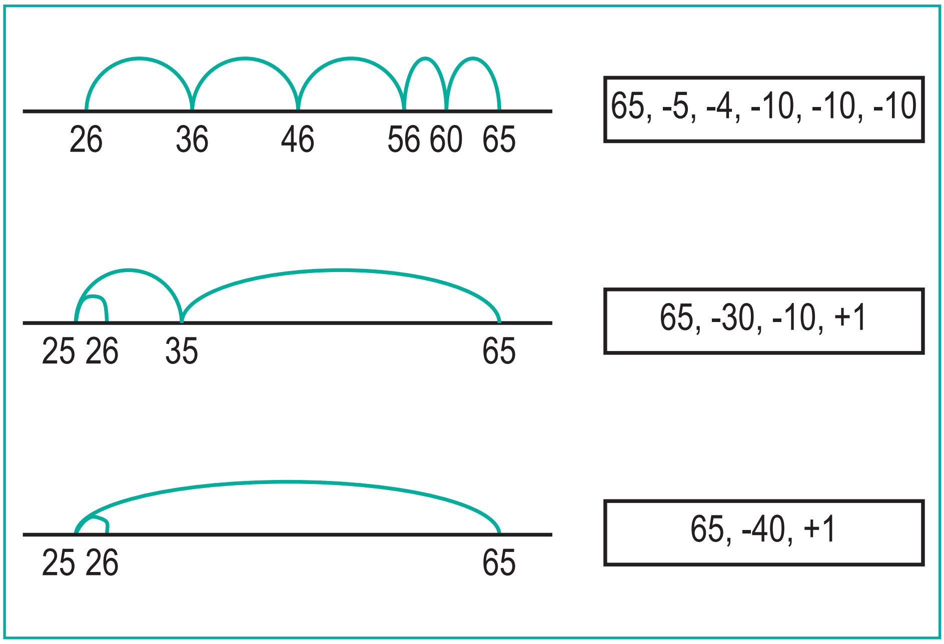

Figure 1. Three solution strategies for 65–39 on the open number line.

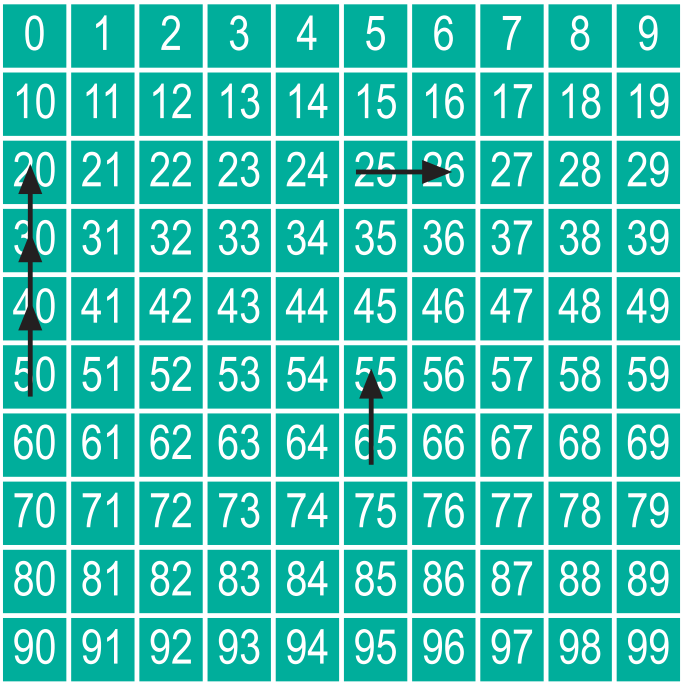

Figure 2. Solution strategy for 65–39 (65–40+1) on the 99 chart.

Multi-base Arithmetic Blocks (MAB) have been used to provide children with a model for mental strategies. However, by the very nature of the material, the use of MAB generally results in the development of less advanced strategies, that is, separation. However, before dismissing MAB as inappropriate material, I would like to cite the case of one Year 3 child who told me he was 'seeing' MAB in his head while he calculated mentally, using quite proficient alternative strategies. He was 'manipulating' the material in such a way as to aid memory during his calculations, and to support his alternative strategies.

Concluding comments

With a renewed emphasis on number sense, it appears imperative that children be encouraged to build on their own natural skills and formulate mental strategies. A classroom environment that promotes freedom to choose, experiment, discuss, and reflect upon strategies will go a long way to fulfilling this aim.

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Ann Heirdsfield teaches at the Queensland University of Technology in Brisbane.

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