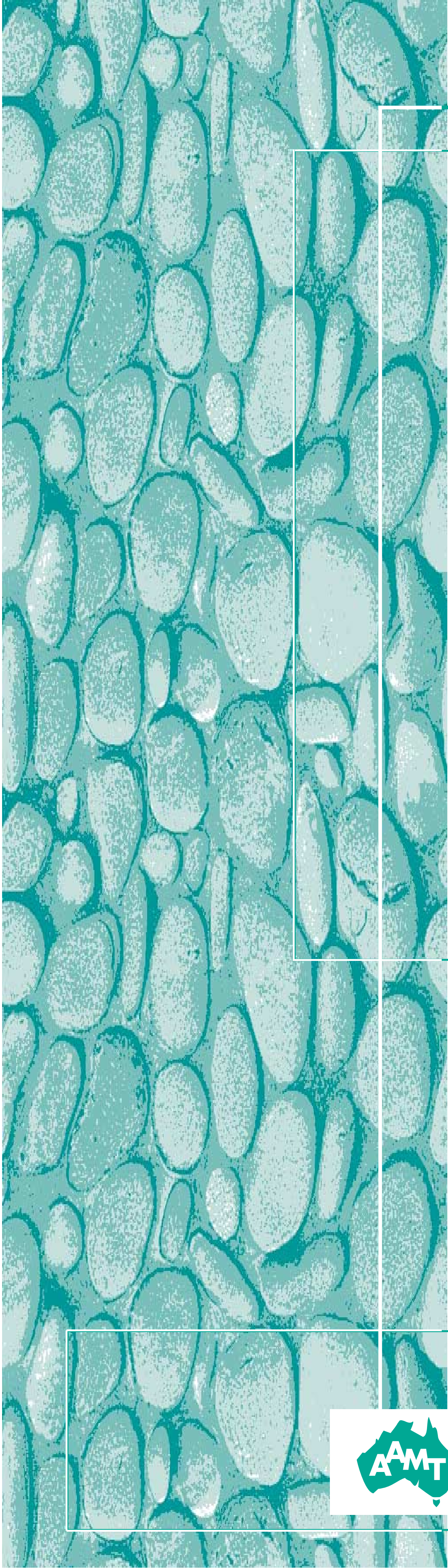


Mathematical Knowledge and Understanding for Effective Participation in Australian Society

1996

The key issue for mathematics education is not whether to teach fundamentals but which fundamentals to teach and how to teach them. Changes in the practice of mathematics do alter the balance of priorities among the many topics that are important for numeracy. Changes in society, in technology, in schools – among others – will have great impact on what will be possible in school mathematics in the next century. All these changes will affect the fundamentals of school mathematics.

A National Statement on Mathematics for Australian Schools

Australian Association of Mathematics Teachers Inc.

NUMBER

The development of number concepts and number skills is probably the most important mathematical requirement of a modern society. It is essential that students develop confidence in their ability to work in number and to deal with numerical situations.

From the study of number, students should:

- develop a rich understanding of number, both of the concept of number itself and of the base ten numeration system used to record and communicate about number.
- become proficient in computation and in making judgments about the application of the four basic operations of addition, subtraction, multiplication and division.
- become familiar with a range of number patterns and relationships.
- develop ready recall of basic number facts and some proficiency in mental computation.
- develop facility in using the equivalent decimal, fraction, and percentage forms of number.
- develop simple techniques for estimating quantities and making quick approximations of calculations.
- develop facility in choosing between mental computation, paper and pen, calculator or computer as the most appropriate method for a particular computation.
- become alert to the need to check the reasonableness of results and to check them in relation to the original situation.
- become confident in using the language of number and in using a variety of notations.

SPACE

A primary goal of mathematics education is to have students develop knowledge about shapes and spaces and their relationships to one another and to the structures, objects and life forms in the wider world. Well developed visualisation skills are a powerful tool in the study of mathematics in general and the ability to visualise is directly applicable to a wide range of occupations, arts and sports.

From the study of space, students should:

- become proficient in identifying basic geometric shapes in two and three dimensions, and their properties, structure and function in design, art and nature.
- become confident in using the language of space and geometry and develop facility in using it to describe, define and classify.
- develop an understanding of symmetry and similarity as basic concepts in design, pattern, scale and technology.
- become familiar with ways of representing location and arrangement, and the conventions for representing them.
- develop the ability to manipulate images mentally (spatial visualisation) and to recognise shapes, arrangements and locations in different orientations.

MEASUREMENT

The application of mathematics in the social, physical, engineering, health and biological sciences, and the humanities often involves measurement. It is important that all students develop an understanding of what, why and how we measure, of the basic principles of measurement, and that they are able to use this knowledge to make sensible judgments about the results of information based on measurement data.

From the study of measurement, students should:

- develop an understanding of the variety and purpose of measurement techniques and tools, and develop facility in selecting the most appropriate unit and instrument for a particular measuring task.
- become proficient in measuring the basic attributes of length, area, volume and capacity, mass, angle, time and temperature.
- become proficient in using standard measures and develop a sense of how much each unit measurement is, together with the ability to estimate quantities in terms of it.
- develop an understanding that measuring involves a decision about precision and level of accuracy.
- develop confidence in using standard formulae for derived measures and known ratios, rates and scales.

CHANCE AND STATISTICS

As the amount and variety of quantitative information presented to and available to the public increases, so does the need to understand the strategies for data collection and analysis, the nature of change processes, and the assumptions that underlie procedures and predictions based on that data. Statistical inference underlies such diverse pursuits as weather forecasting, medical and research design, insurance, gambling and quality control. Students need to clarify their understandings of probability as a measure of chance through engagement in a range of experiments.

From the study of chance and data, students should:

- develop an understanding of chance events and how they are described.
- learn to make sense of, interpret and make judgments about data that has been collected, summarised and presented, by themselves and others, in a variety of ways including tables, charts and graphs.
- develop an understanding of the principles underlying sampling and statistical inference and of the difference between a population and a sample.
- understand how to make predictions from data, examine the assumptions underlying data collection, and question the reasonableness of inferences and conclusions made from such data.
- develop facility in using computer technology to handle data drawn from their experiences and to engage in practical investigations of concern to themselves or their community.

REPRESENTATION OF MATHEMATICAL IDEAS

An important element in any consideration of mathematical knowledge is the ability to communicate in the language of mathematics. With the increasing use of mathematical symbols and visual representations to describe and communicate information about the world and events in it, all students must be able to read and comprehend, write and speak, using appropriate mathematical language. The language of mathematics is concise and precise. Algebra, for example, provides a way to make general statements about patterns in number and space in concise symbolic form.

From a study of the ways mathematical ideas are represented, students should:

- develop facility in reading and comprehending the many ways in which mathematical ideas are communicated.
- develop facility in using symbol systems to represent everyday applications of mathematics.
- develop facility in working with information expressed in symbolic form, particularly ratio, proportion and formulae.
- develop skill in reading and interpreting two-dimensional representations of three-dimensional objects and space.
- develop skill in constructing, reading and interpreting graphs and diagrams that represent relationships and simple functions.
- develop skill in constructing, reading and interpreting tabular, diagrammatic and graphical representations used in the visual presentation of data.
- develop skill in making statements about mathematical situations in a general symbolic form.
- solve equations through understanding and working with relationships between quantities.

It is particularly important that students understand the connections between the relationships and the information (data) that are represented in the very concise symbol system of algebra and that they can interpret a variety of visual representations of that information.

APPLYING MATHEMATICS AND SOLVING PROBLEMS

Many of students' learning experiences in mathematics should stem from solving practical problems in particular contexts through applications familiar to the students. The development of problem solving strategies requires that students have many opportunities to solve a wide variety of problems. Such activities provide opportunities for students to engage in individual and cooperative work and to extend their mathematical thinking through problem posing, reflection and persistence with difficult tasks.

Students need experiences in applying mathematics to practical problems, in acquiring a variety of standard mathematical techniques and choosing appropriately between them, in using non-routine applications, and in using mathematical modelling techniques to solve problems.

The ability to think mathematically is a major component of all activities in the learning of mathematics. No less important is the ability to think mathematically and apply mathematical knowledge, when appropriate, outside strictly mathematical pursuits.

While this statement can function as the basis of a rich and rewarding experience in the learning of mathematics, it must, of its very nature, be limited in scope. It is the responsibility of all teachers of mathematics (and of all teachers who use mathematics in their teaching) to expand students' horizons, confirm their confidence in their own learning and ensure their willingness to continue to learn and use mathematics.

STATEMENTS OF BASIC MATHEMATICAL SKILLS AND CONCEPTS

When the nationally funded Australian Mathematics Education Program (AMEP) was established in 1980, its first task was to develop a 'Statement of Basic Mathematical Skills and Concepts for Effective Participation in Australian Society'. Written in response to a public call for 'back to the basics' in mathematics, the AMEP statement reflected the need for a much broader view of 'basic' mathematics and a more realistic view of the use and application of this mathematics in work, life and society.

In 1980 the statement listed ten areas of mathematics. This was done to extend the public view of numeracy beyond concern with number and measurement. With the development of the National Statement on Mathematics for Australian Schools and the Mayer Report, the public view of numeracy has extended beyond narrow definitions of content. This current statement includes the broad areas of mathematics and of mathematical thinking that are now integral elements of national statements on mathematics for schools in Australia and other nations where English is the first language.

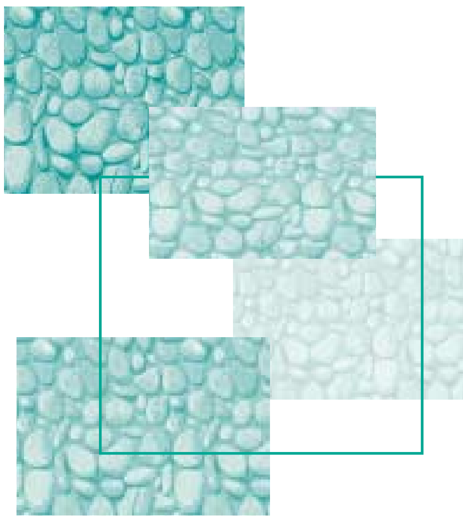
THE NEED TO CHANGE AND EXTEND THE CONCEPT OF NUMERACY

The widespread use of new devices in the information technologies, including calculators and computers, has changed the nature of work for many people and the fabric of social and business interactions for all. It is becoming increasingly important that students have the confidence to use their mathematical knowledge and understandings effectively and that they continue to learn and apply the mathematics they need throughout their lives.

Students need to learn how to engage in working systematically and logically with concrete, abstract and symbolic representations of information and to communicate with others about situations involving mathematical thinking and argument.

It is in the national interest that schools promote high levels of achievement within rigorous mathematics programs, and that as many individuals as possible achieve those levels. All Australian citizens need a broad understanding of mathematics if they are to participate in decision-making about their social and physical environment.

This paper identifies a set of exit learnings for mathematics that should be built up through mathematical experiences during the years of schooling. Mathematics is an integral part of general education. It enhances understanding of the world and of how number, space, measurement, pattern and chance are used and how they affect everyday lives. Mathematical understandings are important in our personal, vocational and civic lives, and they underpin most other branches of human learning, such as the sciences, social sciences, arts, business, trades, design and further mathematics itself.



MATHEMATICS AND SOCIETY

Any statement on mathematics education must be unique to the society in which that education is to occur. This statement is directed specifically to the Australian context. Australia is a multicultural society and, for some groups, some of the knowledge and understandings that are goals for learning in this statement may be difficult to acquire because of language, locale, or other differences including access to technology.

Mathematics, like all subjects taught in schools, is culturally biased both at the national and international level. The usual medium of instruction in Australia is English and this allows sharing of the heritage of English mathematical language and notations. There is an added richness in mathematics classrooms where students from different backgrounds share the mathematics from their cultures.

This statement also serves to draw attention to the use of mathematics tests as a screening device by employers and institutions. To the extent that these tests do not reflect the learnings outlined in this paper they serve to distort the school curriculum.

Some further reasons for preparing this statement are:

- to provide a national document so that discussion and debate about mathematics education can be based on shared understandings and can therefore be productive and meaningful.
- to alert employers to the changing mathematical needs, demands and dependency of Australian society and to alert teachers to the implications of these changes for school programs.
- to inform the development of an appropriate benchmark by which employers can judge the mathematical competence of their employees for initial training.
- to provide a basis for evaluating the policies, test instruments and interpretive reports undertaken by state and national agencies.
- to draw attention to the increasing use of and dependence on mathematics and mathematical thinking by the community at large.
- to strengthen the commitment to real and realistic applications of school mathematics, drawn from commerce, industry and life.
- to draw the attention of educators and the public to the unique relationship between language, communication and mathematics, in particular the role of symbols in the learning of mathematics.

Discussion points

- In what ways does this statement need to be amended to reflect the special needs of students from different educational and cultural backgrounds?
- In addition to providing 'fundamental mathematical understandings', school mathematics programs should prepare students for further study in mathematics. How should this list be extended to reflect the needs of students who wish to pursue further extensive study in mathematics?
- Although the use of technology is mentioned, facility in using technology itself is not seen as a component of a mathematics curriculum alone, but as a shared responsibility across the whole school curriculum.

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