Complete these proofs, putting in the reasons and missing angles. Mark the equal angles and sides you find on the diagram as you go.

1. **Given:** \(BC = DC; AB \perp BC \text{ and } CD \perp DA\)  
   **Aim:** To prove \(\triangle ABC \cong \triangle ADC\)  
   **Proof:**  
   In \(\triangle ABC \text{ and } \triangle ADC\)  
   1. \(\angle B = \angle D = 90^\circ\)  
      (given)  
   2. \(AC\) is common  
      (given)  
   3. \(BC = DC\)  
      \(\therefore \triangle ABC \cong \triangle ADC\)  
      (R H S)

2. **Given:** \(ABCD\) is a rectangle  
   **Aim:** To prove \(\triangle ABC \cong \triangle ADC\)  
   **Proof:**  
   In \(\triangle ABC \text{ and } \triangle ADC\)  
   1. \(AB = DC\)  
      (opposite sides rectangle)  
   2. \(AD = BC\)  
      (as above)  
   3. \(AC\)  
      \(\therefore \triangle ABC \cong \triangle ADC\)  
      (S S S)

3. **Given:** \(AB \parallel DC \text{ and } BP = PD\)  
   **Aim:** To prove \(\triangle ABP \cong \triangle CDP\)  
   **Proof:**  
   In \(\triangle ABP \text{ and } \triangle CDP\)  
   1. \(\angle A = \angle C\)  
      (alternate angles; \(AB \parallel CD\))  
   2. \(\angle APB = \angle DPC\)  
      (vertically opposite)  
   3. \(BP = PD\)  
      (given)  
      \(\therefore \triangle ABP \cong \triangle CDP\)  
      (A A S)
4. Aim: To prove $BD = DC$

Proof:
In $\triangle ABD$ and $\triangle ACD$
1. $\angle ADC = \angle ADB = 90^\circ$ (given)
2. $AB = AC$ (given)
3. $AD = AD$ (common)
   $\therefore \triangle ABD \equiv \triangle ACD$ (RHS)
   $\therefore BD = CD$ (matching sides of congruent Ds)

You cannot use the property of $AD$ bisecting $BC$, as this is the goal of the question!

5. Aim: To prove $PQ \parallel ST$

Proof:
In $\triangle PQR$ and $\triangle STR$
1. $PR = RT$ (given)
2. $\angle PRQ = \angle SRT$ (vertically opposite)
   $QR = RS$ (given)
   $\therefore \triangle PQR \equiv \triangle TSR$ (SAS)
   $\therefore \angle PQR = \angle RST$ (matching angles of congruent Ds)
      But these are alternate angles
   $\therefore PQ \parallel ST$ (alternate angles are equal)

6. Aim: To prove $\angle B = \angle D$

Proof:
In $\triangle ABC$ and $\triangle ADC$
1. $DC = AB$ (given)
2. $\angle BAC = \angle ACD$ (alternate angles; $AB \parallel CD$)
3. $AC$ is common
   $\therefore \triangle ABC \equiv \triangle ADC$ (SAS)
   $\therefore \angle B = \angle D$ (matching angles of congruent Ds)
7. Given: \( PQRS \) is a parallelogram. \( PT = RU \).
Aim: To prove \( TS = QU \)

In \( \triangle PTS \) and \( \triangle RUQ \)
1. \( PT = UR \) (given)
2. \( \angle TPS = \angle QRS \) (opp. angles parallelogram)
3. \( PS = QR \) (opp. sides parallelogram)
\( \triangle PTS \equiv \triangle RUQ \) (SAS)
\( \therefore TS = QU \) (matching sides of cong. triangles)

8. Given: \( ABCD \) is a square. \( BH \perp AP \) and \( DK \perp AP \).
Aim: To prove \( AH = DK \)

Proof:
In \( \triangle ABH \) and \( \triangle ADK \)
1. \( \angle AHB = \angle AKD = 90^\circ \) (given)
2. \( \angle HAB + \angle ABH + \angle AHB = 180^\circ \) (angle sum \( \triangle AHB \))
\( \therefore \angle ABH = 90^\circ - \angle HAB \)
   But \( \angle DAK = 90^\circ - \angle HAB \)
   (\( \angle DAB = 90^\circ \), \( ABCD \) is a square)
\( \therefore \angle ABH = \angle DAK \)
3. \( AB = AD \) (sides of a square)
\( \therefore \triangle ABH \equiv \triangle DAK \) (AAS)
\( \therefore AH = DK \) (matching sides of cong. triangles)

9. Given: \( ABCD \) and \( AEFG \) are both squares.
Aim: To prove \( BE = DG \)

Let \( x = \angle EAB \)
\( \therefore \angle BAG = 90^\circ - x \) (\( \angle EAG = 90^\circ \), square \( EAGF \))
\( \therefore \angle GAD = 90^\circ - (90^\circ - x) = x \)
   (\( \angle BAD = 90^\circ \), square \( ABCD \))
In \( \triangle AEB \) and \( \triangle AGD \)
1. \( AB = AD \) (sides of square \( ABCD \))
2. \( \angle EAB = \angle GAD \) (see above)
3. \( AE = AG \) (sides of square \( AEFG \))
\( \triangle AEB \equiv \triangle AGD \) (SAS)
\( \therefore BE = DG \) (matching sides of cong. triangles)