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Put Mental computation first?

Arguments about the place of standard methods, the calculator and mental mathematics abound. Geoffrey Morgan adds to this debate by proposing a framework for computation.

Introduction

A focus on mental computation is critical to a revitalisation of school mathematics. For school mathematics to be useful, it needs to reflect the computational techniques used in everyday life. Whereas school mathematics continues to be oriented towards standard paper and pencil techniques, those techniques used outside the classroom are predominantly mental. Such methods are closely intertwined with the features of the context of an arithmetical problem to be solved.

Mental and written computation

Bridging the gap between computational techniques used within the classroom and those used beyond is critical to students developing confidence in their mathematical abilities. Those who are proficient at mathematics in daily life, including the workplace, seldom make use of the standard computational techniques taught in schools. Rather, idiosyncratic methods tend to be used or else unique adaptations of the written algorithms are developed. As an outcome, researchers, mathematics educators and policy makers have advocated that the current emphasis on the standard paper and pencil algorithms needs to be reduced. Such a reduction is essential to dispelling the erroneous view of arithmetic as essentially involving linear, precise, and complete calculations.

However, the impact of an emphasis on mental computation would not be limited to its social utility. Such an emphasis would significantly contribute to the development of number sense through fostering ingenious ways in which to manipulate numbers. Number sense depends upon, and contributes to, the development of a deeper understanding of the structure of numbers and their properties. Further, mental computation is viewed as an essential prerequisite to the successful development of written algorithms. It is the concern for these aspects of mental computation that Reys and Barger

(1994) believe to be the novel facet of the current interest in mental computation; one which highlights mental computation as a means for promoting thinking, conjecturing, and generalising based on conceptual understanding.

Nevertheless, despite the ongoing advocacy for an increased emphasis on teaching rather than testing mental computation, this has yet to significantly translate into classroom practice. The unwillingness on the part of many students to attempt to calculate mentally and the concomitant low standard of mental computation, need to be overcome. Children need to be encouraged to value all methods of computation and particularly to develop personal strategies for calculating mentally. Teachers need to come to recognise the legitimacy of the development of mental skills as a major goal for school mathematics, and in so doing change the way in which mental computation is viewed.

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Sequences for introducing computational procedures

The research literature relating to the most appropriate ways to incorporate mental computation into the curriculum is characterised by a degree of equivocalness. McIntosh (1998) has cautioned that 'helping children to acquire ownership of efficient strategies is not likely to be achieved by "teaching" these strategies at the expense of understanding' (p. 221). Nonetheless, irrespective of the approach employed, teaching to promote the desired outcomes occurs within an overall sequence for introducing computational procedures. This sequence may be considered at two levels: (a) the order in which mental, technological, and paper and pencil techniques are taught to children; and (b) the order in which particular procedures associated with each of these methods are learned. Of these, the first may have greater significance for mental computation.

Traditional sequence for introducing mental and written computation

The written mental sequence in which computational procedures have usually been introduced (Figure 1) is inextricably bound with the purposes for learning school mathematics. Although current syllabuses exhibit an emphasis beyond arithmetic, the prime focus of primary school mathematics continues to be written computation and the development of standard written algo-

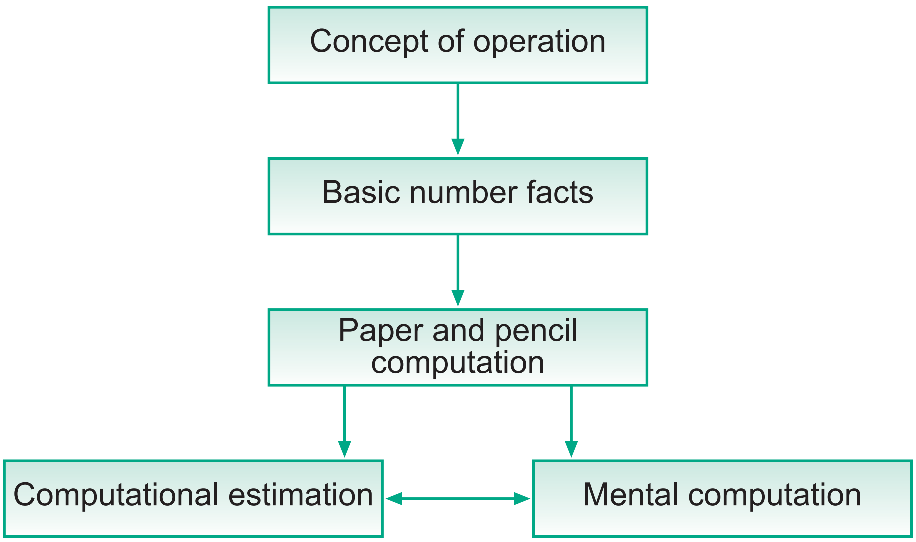


Figure 1. Traditional sequence for introducing computational procedures for each operation (Adapted from Irons, 1990a)

Written algorithms are products of the needs of an industrial age

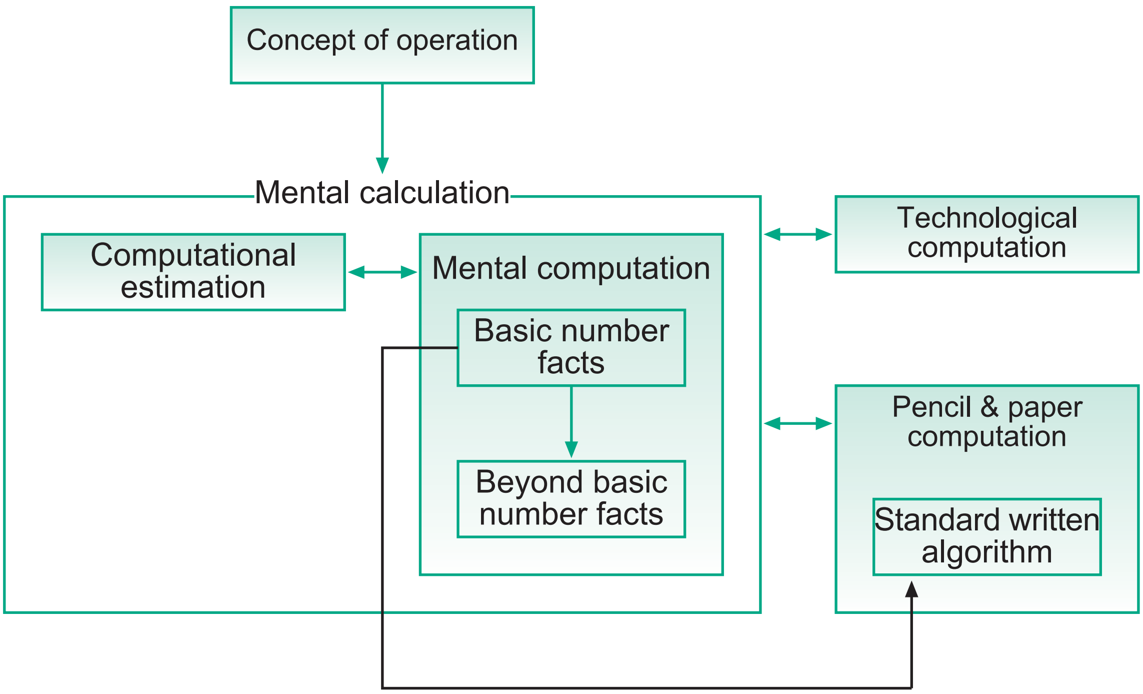
rithms. The goal continues to be the automatic processing of paper and pencil calculations, despite the concern for developing students' understanding of the processes involved.

Written algorithms are products of the needs of an industrial age that necessitated minimum competencies in arithmetic for all students, with higher mathematical training reserved for a few. The continued emphasis on written procedures has its origins in two outdated, but steadfastly held, assumptions, namely that (a) mathematics is a fixed set of facts and procedures, and (b) using mathematics entails calculating answers to given problems through applying specified techniques. The 'long-standing preoccupation with [written] computation... has dominated what mathematics is taught and the way mathematics is taught' (National Council of Mathematics Teachers, 1989, p. 15). One consequence of this has been that a student's view of mathematics has generally not reflected the subject's vitality, an essential element in promoting the development of flexible mathematical thinking.

Alternative sequence for Introducing Mental and Written Computation

Critical to flexible mathematical thinking is an individual's sense of number, the development of which is considered by the Australian Education Council (1991) to be an essential goal for primary school mathematics. Number sense is, in part, characterised by an ability to perform mental computations with nonstandard strategies that take advantage of an ability to compose and decompose numbers. In so doing, students with number sense tend to analyse the whole problem first to ascertain and capitalise upon the relationships among the numbers, and the operations and contexts involved, rather than merely apply a standard algorithm.

The development of flexible mental strategies is influenced by the order in which mental and written techniques are introduced. Classroom experience indicates that children have difficulty with mental methods when written algorithms are taught prior to a focus on mental computation. Such a focus places an emphasis on symbols rather than on the quantities



Note: using school-authorised and idiosyncratic procedures

Figure 2. An alternative sequence for introducing computational procedures for each operation (Morgan, 2000)

embodied in calculative situations, thus reducing the opportunities for the development of number sense. Further, the right to left characteristics of the standard written algorithms for addition, subtraction, and multiplication contradict the holistic, left to right strategies frequently used by proficient mental calculators. Hence, the conventional written before mental sequence needs to be re-evaluated. One consequence of such a re-evaluation would be the development of more flexible computational strategies, both mental and written.

Figure 2 presents an alternative view of the sequence for developing computational skills. In implementing this sequence, recognition needs to be given to the role of mental computation as a fundamental component of computational estimation. Therefore, the development of selected strategies for mentally calculating exact answers needs to occur prior to a focus on computational estimation. As Sowder (1992) has noted, a focus on computational estimation should not occur too soon after the introduction of a particular operation. Rather, teachers should focus on developing number size concepts and mental computation strategies, and on estimation-type problems that do not require the coordination of complex skills.

The place of standard written algorithms in the mathematics curriculum

Even as far back as 1984, Reys suggested that standard written algorithms discourage thinking. They are designed to be used automatically by students who require only a limited understanding of the processes involved. Hence, they contribute little to the development of number sense, particularly where decontextualised examples are presented to students.

If standard paper and pencil algorithms are introduced after an initial focus on strategies for calculating both exact and approximate answers mentally, and following experiences with informal written techniques, it is likely that a standard algorithm for each operation will come to be viewed as one of many possible ways for calculating in particular contexts, rather than the way to calculate. The ability of children to make choices between calculative methods will be enhanced — an essential computational outcome for classroom mathematics programs.

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A sequential framework for introducing mental, calculator and written computation

Table 1 offers a revised sequential framework for mental, written and calculator procedures. It attempts to put into practice the models shown in Figure 2. Here, an emphasis is placed on the use of mental and calculator procedures for each operation beyond the basic facts prior to the introduction of paper and pencil techniques. It assumes the availability of calculators for computations beyond those capable of being worked mentally, thus maximising opportunities for their becoming real computational tools, particularly for low-attaining children.

Mental strategies (see Table 1) refers to strategies for calculating both approximate and exact answers. Such strategies take advantage of the structural properties of numbers and the relationships between them. Their development and that of number comparison and number sense occurs spirally, each ‘feeding on and strengthening the others’ (Threadgill-Sowder, 1988, p. 195). The ability to compute mentally with truncated and rounded numbers is a prerequisite for computational estimation. Additionally, mental strategies used to refine estimates may assist in the development of flexible approaches for calculating exact answers; getting closer may ultimately result in turning approximate answers into exact answers.

In concert with the analysis presented above, the emphasis for written procedures is placed on self-generated strategies. However, reference to a standard written algorithm for each operation has been retained. From personal observations of previous syllabus changes, the reality of the classroom dictates that they will continue to

Table 1

Revised sequential framework for introducing mental, calculator and written procedures for addition, subtraction, multiplication and division.

	Year		
	1–3	3–5	5–7[a]
ADDITION			
1. Addition concept	✓	✓	
2. Non-written procedures	✓	✓	
(a) Addition facts — thinking strategies	✓	✓	
(b) Addition beyond basic facts	✓	✓	
• Self-generated/ shared mental strategies	✓	✓	
• Calculator strategies	✓	✓	
3. Written procedures	✓	✓	
• Self-generated/shared strategies	✓	✓	
• Standard written algorithm		✓	
SUBTRACTION			
1. Subtraction concept	✓	✓	
2. Non-written procedures	✓	✓	
(a) Subtraction facts — thinking strategies	✓	✓	
(b) Subtraction beyond basic facts	✓	✓	✓
• Self-generated/ shared mental strategies	✓	✓	✓
• Calculator strategies	✓	✓	✓
3. Written procedures		✓	✓
• Self-generated/ shared strategies		✓	✓
• Standard written algorithm			✓
MULTIPLICATION			
1. Multiplication concept	✓	✓	✓
2. Non-written procedures	✓	✓	✓
(a) Multiplication facts — thinking strategies	✓	✓	✓
(b) Multiplication beyond basic facts		✓	✓
• Self-generated/ shared mental strategies		✓	✓
• Calculator strategies		✓	✓
3. Written procedures		✓	✓
• Self-generated/ shared strategies		✓	✓
• Standard written algorithm			✓
DIVISION			
1. Division concept		✓	✓
2. Non-written procedures		✓	✓
(a) Division facts — thinking strategies		✓	✓
(b) Division beyond basic facts		✓	✓
• Self-generated/ shared mental strategies		✓	✓
• Calculator strategies		✓	✓
3. Written procedures			✓
• Self-generated/ shared strategies			✓
• Standard written algorithm[b]			✓

Note:
(a) Year 7 is a primary class in Queensland.
(b) Introduce using current sequences.

be taught, even should a revised syllabus advise otherwise. The degree to which standard algorithms will continue to be taught is dependent upon the effectiveness of the professional debate, supported by further research into managing mathematics classrooms in which the focus is on self-generated mental, technological, and written strategies. 'Our challenge as educators is to identify what is being learned from the algorithm (whether it be traditional or not) besides the ability simply to execute it. History shows that relinquishing this tradition of "algorithms for algorithm's sake" will be difficult' (Barnett, 1998, p. 77).

Most importantly, the focus on non-written and non-standard written procedures may assist in overcoming the belief of many adults and students, as identified by Plunkett (1979), that the concept of a particular operation and its standard paper and pencil algorithm are synonymous. To support this focus, the placement of the standard written algorithms within the sequence accords with the recommendation of the Australian Education Council (1991) that, in instances where these algorithms continue to be taught, such teaching should occur later in a child's schooling.

The framework presented in Table 1 recognises that schools need to acknowledge that all students can learn, and to focus on differences with respect to the way students learn and the rate at which learning occurs. Such a focus requires a flexible approach to the allocation of particular learnings to particular time periods. Hence, the elements of the sequential framework are not year level specific. It is intended that the overlapping bands provide for a smooth progression through the learnings related to computation, while recognising that each class is characterised by students who exhibit a range of achievement levels.

Conclusion

The introduction of curricula in which mental computation and computational estimation skills are introduced earlier, in combination with a ready availability of electronic calculating devices, will fundamentally change the manner in which computation is taught in primary classrooms. These changes could be expected to address issues central to teachers' beliefs about the nature of school mathematics and mathematics teaching. It is therefore essential that debate, both professional and community, occurs well in advance of the implementation of curriculum documents. Such debate would provide an understanding of, and assist in the development of a commitment to, the recommended changes. Moreover, it would serve to preserve the sense of efficacy teachers exhibit in their role as teachers of mathematics in the primary school.

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Editors' note: references to support Geoffrey's discussion can be obtained from his website at:
www.ozemail.com.au/~gmorgan/mc_ref

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