Document extract

Title of chapter/article	Is Your Classroom Mental?
Author(s)	Emilia Mardjetko & Julie Macpherson
Copyright owner	The Australian Association of Mathematics Teachers (AAMT) Inc.
Published in	Australian Primary Mathematics Classroom vol. 12 no. 2
Year of publication	2007
Page range	4–9
ISBN/ISSN	1326-0286

AAMT—supporting and enhancing the work of teachers

The Australian Association of Mathematics Teachers Inc.
ABN 76 515 756 909
POST GPO Box 1729, Adelaide SA 5001
PHONE 08 8363 0288
FAX 08 8362 9288
EMAIL office@aamt.edu.au
INTERNET www.aamt.edu.au

Is your classroom mental??

Guidelines for enhancing the development of strategies for mental computation


Emilia Mardjetko and Julie Macpherson put a strong case for an emphasis on developing mental calculation strategies with students and suggest helpful teaching approaches to achieve this.

Without pen, paper or calculator how would you work out the answer to this question - 46+68? Now take 15 away from 32.

What methods did you employ? For the addition question did you first add 40 and 60 together then add six plus eight and combine the 100 with 14–just like Steve in Figure 1? This method of splitting both numbers, using knowledge of place value, and then adding the parts of each number working from left to right (tens before units) is referred to as left to right separated place value. Perhaps you used Laura's strategy (Figure 1), leaving 68 whole, then adding 40 then 6. In this case you have used the left to right aggregation method, which also has a right to left mode. Maybe you simply visualised the vertically written algorithm and carried or borrowed tens. What strategy would students in your classroom utilise? Would they count between, eg. count on ten six times and then add eight, round up or down similar to Jacqui's approach (Figure 1) or utilise a levelling approach like that of David (Figure 1). The four mental computation strategies shown in Figure 1 may be similar to those you have seen utilised by your students.

You may have also observed students using alternative methods.

It is important to consider the accuracy and efficiency of various mental computation strategies and this can be useful to discuss with students. Some strategies may result in errors. For example, when working out the addition question given in Figure 1 it is likely that some students may have place value errors resulting in confusion between tens and ones. Working out the subtraction question may result in a "smaller from larger bug" error if 32 – 15 = 23 was given as the response (Heirdsfield, 2004).

Out of curiosity ask other colleagues and family members what strategies they would employ. What did you discover from your research? It is likely that a range of strategies were used. Try this activity in your own classroom and have a discussion with your students which focuses on the strategies they used. Discussion about different strategies will enable students to consider different approaches and work towards development of efficient mental strategies for computation. Utilising a variety of strategies and methods for computation will enable students to develop a better understanding of computation processes and number sense (Reys, 1985).

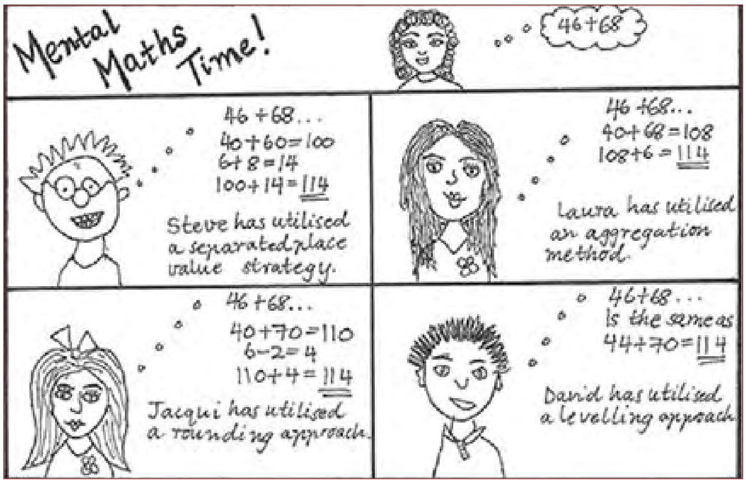

Figure 1: Addition question - sample strategies for mental computation

McIntosh (2002, 2005) commented on the change in focus of recent Australian Federal and State Government education policy documents with a shift away from students being taught algorithms for all mathematical computations (which includes both written and mental computation), to a policy centred on development of a variety of strategies for mental computation. Kamii (1994) also suggested widespread support for mental computation in the belief that an early emphasis on learning algorithms was a mathematical health hazard that inhibits children's own numerical thinking, retarding development of number sense and adding to children's confusion with place value.

Bebout (1990) reported that very young children have effective strategies for mental computation for solving basic addition and subtraction problems. These pre-school strategies for mental computation rely on modelling or counting processes and continue to develop beyond preschool (Carpenter & Moser, 1982, 1983, 1984; Hiebert, 1982, cited in Bebout, 1990). A two-year Queensland study of Grade 2 and 4 children found the number of children utilising the counting strategy decreased over time, however it was still in use by some of the lower scoring students (Cooper, Heirdsfield & Irons 1996).

What is mental computation?

Mathematical computation consists of both written computation and mental computation. The strategies for mental computation can be used to check the reasonableness of written computations. Mental computation has two distinguishing characteristics; "it produces an exact answer, and the procedure is performed mentally, without using external devices such as pencil and paper" (Reys, 1984, p. 548).

Mental computation provides a valuable and useful connection between problem solving and mathematical concepts but the principal focus of mathematical computation in the primary school has been the written pen and paper algorithms. These written algorithms impact on mental computation and a common strategy used for mental computation is visualisation of the written pen and paper algorithm (see Lynda & Gloria in Figure 2). Utilising this strategy of visualising a written algorithm can be prone to error and shows little number sense. In Figure 2, Kaye demonstrates a good understanding of number relationships to solve the problem 32-15.

Strategies for mental computation and the development of number sense

McIntosh (2004) found that up to 20% of upper primary students continue to use counting by ones for two digit addition and subtraction questions. In a Victorian study of Grade 3-5 classes where students were tested with mathematical items for which they had not been taught algorithms, two major types of strategies were observed. Attempting to obtain an answer purely through visualization of the written pen and paper algorithm like Lynda and Gloria (Figure 2) was the least successful strategy, whereas students who understood the question, like Kaye, were able to manufacture legitimate and effective strategies for mental computation (Mackinlay, 1996).

Kamii, Lewis, and Jones, (1991) believe that rote learning of written pen and paper algorithms by children places the focus of learning on the algorithm rather than the development of number sense.

Algorithms may reinforce the concept that every column in a written place value is a units or ones column.

(Resnick, 1986; Wearne, 1990 cited in Markovits & Sowder, 1994) argue that the development of place value and number sense requires an understanding of the relationship between two numbers.

Hope and Sherrill (1987) contend that several factors impact on an individual's ability to demonstrate strategies for mental computation. These include their available strategies, number relationship knowledge and number manipulation skills. Hitch (1977, 1978 and Merkel & Hall, 1982, cited in Hope & Sherrill, 1987) believes that the most effective mental computation strategies are those that sequentially build up to the answer. For example, refer to the variety of strategies for mental computation utilised by the students in Figure 1.

Research has shown a targeted program can result in a rapid improvement in the development of strategies for mental computation.

A Queensland study of one Year 3 class incorporated a ten-week program of teaching mental computation strategies for two and three digit addition and subtraction and included pre and post interviews. The focus of this study was to determine what issues impacted on mental computation performance and the development of higher order thinking. In addition to the quality of lessons and tasks, a major factor involved the establishment of connections and encouragement of strategic thinking. These connections and strategic thinking practices were developed by a well-planned teaching sequence of activities and follow up discussions.

Following the study, students were re-introduced to written pen and paper algorithms and were noted to approach these with better understanding (Heirdsfield, 2005). Successful mathematical instruction develops flexibility, exploration and justification of strategies by students (Kamii & Dominick, 1998 cited in Heirdsfield, 2005). Reys (1985) believes there are solid reasons supporting the development of mental computation strategies and these relate to the fact that as daily mathematical transactions become more automated and computerised there is less of a requirement to perform written pen and paper algorithms. There is a need however,

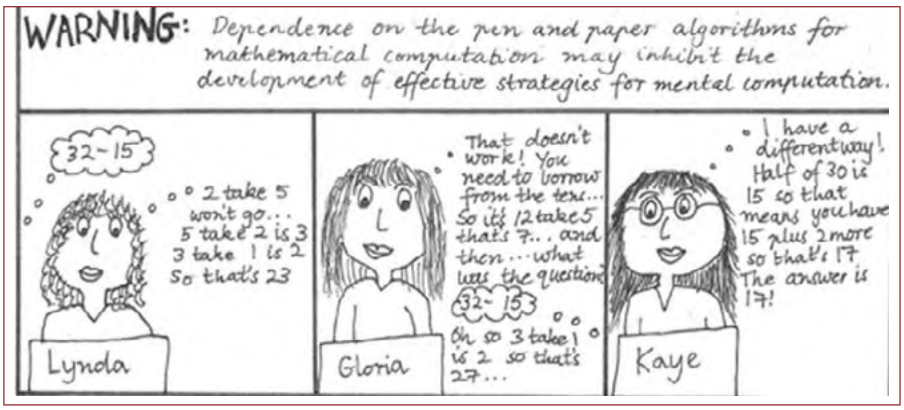

Figure 2: Subtraction question - sample strategies for mental computation.

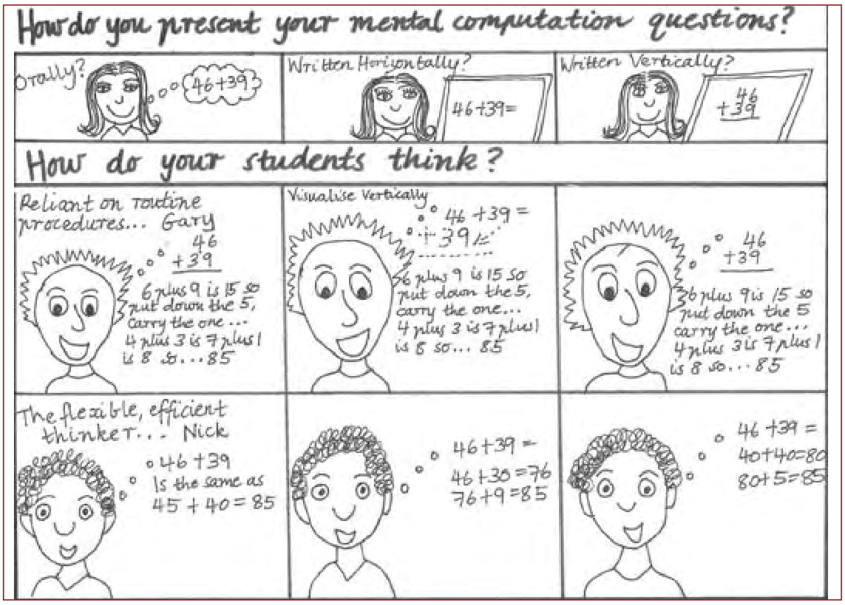

Figure 3: Various modes of presentation – sample strategies for mental computation.

for individuals to have the strategies for mental computation and estimation skills to be able to check these automated calculations.

Mode of presentation can affect strategy use and accuracy

Mathematical questions can be presented in a variety of modes. This includes oral, vertical and horizontal, as well as items in context. In some classrooms, items are presented orally by the teacher and in other cases students work from material visually presented on the board or in a textbook. In many cases, both methods are used interchangeably, with no consideration given as to whether the mode of presentation affects students' performance. Research has indicated that the mode of presentation of items can affect both student performance on mental computation and the choice of mental computation strategy. Visualisation of written pen and paper algorithms resulted in higher error rates, with this strategy used least by higher performing students (Reys, Reys, Nohda & Emori 1995). One explanation of the errors made when visualising the written pen and paper algorithms is that the 'carry' operation can be quite problematic for mental imaging, refer to Gloria in Figure 2 (Hitch, 1977, 1978; Merkel & Hall, 1982, cited in Hope & Sherrill, 1987).

We have observed that mode of presentation of items does in fact affect the performance of students. We will now outline some different modes of presentation and possible effects on students' ability to correctly perform mental computation. The problem 46 + 39 is presented three different ways in Figure 3. The first example has the teacher presenting the problem orally, the second has a horizontal presentation and the final example is written vertically.

Commonly, students as Gary in Figure 3, utilise the strategy which is similar to that of the written pen and paper algorithm. For the orally presented question, Gary visualises the steps he would perform using pen and paper. When the problem is written horizontally he visualises the problem vertically to perform the mental calculation.

When performing mental calculation place value may be ignored. Numerals may be separated with students moving from right to left, as Gary has in Figure 3. However, other students who use this strategy may move from left to right when presented with oral items. It is argued that higher performance on orally presented items may indicate success in applying more flexible mental strategies (McIntosh, Nohda, Reys & Reys, 1995).

Nick (Figure 3) is an example of a student who has a range of strategies for dealing with mental computation. Good number sense enables him to choose efficient strategies for performing mental computation in a number of ways.

These findings support the inclusion of various modes of presentation within your teaching program to ensure that a wide variety of strategies are explored and developed. Importantly, students must discuss their own spontaneous strategies with one another so that a broad range of strategies can be recognised and considered for future use.

Teaching implications

If we want students to develop good strategies for performing mental computations then it is important to consider how the development of strategies might be incorporated into the classroom. This is important to promote as:

- pen and paper abilities do not correlate to mental computation abilities,
- dependence on visualisation of the written algorithm relies on good short term memory and accurate number facts.
- individual assessment (teacher – student interview) is required in order to determine students' mental computation abilities with additional discussion on strategy application.

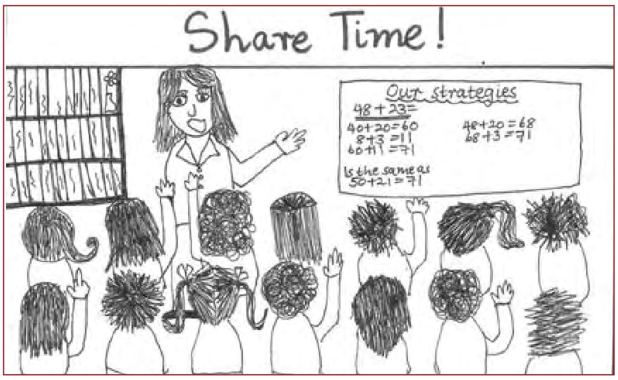

Figure 4: Students sharing strategies for mental computation.

Principles for ensuring a mental classroom!

1. Incorporate class discussions to allow students to share and model a variety of strategies for mental computation to develop confidence in their ability to try alternative strategies for solving questions.
2. Delay formal teaching of pen and paper algorithms until students have flexible mental computation strategies.
3. Accept and acknowledge students' spontaneous and creative strategies. Be open to these strategies.
4. Promote the importance of mental computation by conducting a structured program to build and develop skills with a particular focus on checking the reasonableness of answers.
5. Provide students with a range of questions which are embedded within real life experiences.
6. Provide students with various modes of presentation including oral, horizontal and vertically presented questions.

Conclusion

Development of strategies for mental computation is extremely important for primary school students. Students should be given opportunities to build their own strategies for mental computation, while verbalising and verifying the appropriateness of the strategies applied. Less emphasis should be placed on the teaching of the written pen and paper algorithm and more emphasis should be directed towards the identification and development of students' spontaneous strategies for mental computation. Generally, students who have greater flexibility in their mental computation strategies provide a greater understanding of the underlying mathematical concepts and have higher success rates.

Activities conducted within the classroom should encourage effective and spontaneous strategies and these will form the basis for understanding mathematical processes and concepts.

Resources:

1. McIntosh, A. (2004). *Mental computation: a strategies approach. Hobart: Department of Education* (available from the Tasmanian Department of Education for $28 including P&P).

This folder contains a step-by-step development of mental computation and estimation skills for all primary school levels and documents research and grading scales of skill achievement.

2. Stacey, K., Varughese, N. & Marston, K. (2003) *Teaching mental and written computation* CD, University of Melbourne. (an online sample can be viewed at – http://extranet.edfac.unimelb.edu.au/DSME/tmwc/index.shtml; (available from AAMT office)

This CD includes:

- mental methods used by children including QuickTime movies
- explanations of common errors
- teaching strategies, activities, work sheets and diagnostic tests.

References

Bebout, H. (1990). Children's symbolic representation of addition and subtraction word problems. *Journal for Research in Mathematics Education, 21*,(2), 123–131.

Cooper, T., Heirdsfield, A. & Irons, C. (1996) Children's mental strategies for addition and subtraction word problems) In J. Mulligan & M. Mitchelmore (Eds) *Children's Number Learning.* (pp147–162). Adelaide: The Australian Association of Mathematics Teachers Inc.

Heirdsfield, A. (2004) Inaccurate mental addition and subtraction: causes and compensation. *Focus on Learning Problems in Mathematics Vol 26* (3),43–65.

Heirdsfield, A. (2005) One teacher's role in promoting understanding in mental computation. In H. Chick & J. Vincent (Eds). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3*, pp. 113–120. Melbourne: PME.

Hope, J. & Sherrill, J. (1987) Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education, 18*(2), 98–111.

Kamii, C. (1994). Young Children Continue to Reinvent Arithmetic – 3rd grade: Implications of Piaget's Theory. New York: Teachers College Press.

Kamii, C., Lewis, B. & Jones, S. (1991) Reform in primary mathematics education: A constructivist view. *Education Horizons, Fall*, 19–26.

Mackinlay, M. (1996) *Children's Informal Written Computation Methods*. Paper presented at the joint conference of the Educational Research Association (ERA) and the Australian Association for Research in Education (AARE), Singapore, 25–29 November, 1996. Available: http://www.aare.edu.au/96pap/makn96177.text

McIntosh, A. (2002) *Developing Informal Written Computation*. Paper presented at the Annual Conference of the Australian Association for Research in Education, University of Queensland, Brisbane, December 2002. Available: http://www.aare.edu.au/02pap/mc102517.htm

McIntosh, A. (2004). *Mental Computation: A Strategies Approach. Module 1 - Introduction.* Hobart: Department of Education.

McIntosh, A. (2005). *Developing Computation*. Hobart: Department of Education, Tasmania.

Markovits, Z. & Sowder, J. (1994). Developing number sense: an intervention study in Grade 7. *Journal for Research in mathematics Education, 25*. No 1, 4-29.

Reys, R. E. (1984). Mental computation and estimation: Past, present and future. *Elementary School Journal, 84*, 546-557.

Reys, B. (1985) Mental computation. *Arithmetic Teacher, 32*(6), 43-46.

Reys, R., Reys, B., Nohda, N. & Emori, H. (1995) Mental computation performance and strategy use of Japanese students in grades 2,4,6 and 8. *Journal for Research in Mathematics Education, 26*(4), 304-326.

Emilia Mardjetko
University of Melbourne
<mardjetko@edumail.vic.gov.au>
Julie Macpherson
University of Melbourne
<mc@aia.vic.edu.au>

APMC