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"a journal to serve as a medium both for the exchange of ideas and experiences in the teaching of elementary mathematics and for the instruction of teachers in the trends and developments of mathematics education at home and abroad"
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DISCOVERY

with Neville de Mestre

Crossing the desert

Here is a logic problem which will test your students' ability to construct useful diagrams as an aid to discovering the solution.

A truck can carry at most five drums of fuel and another two drums can fill its fuel tank. It has to cross a desert which is 1000 km wide and there are no fuel stations in the desert. Fortunately there are an unlimited number of drums of fuel at the edge of the desert from which the truck departs. Each drum contains enough fuel to allow the truck to travel 100 km.

Ask your students to plan the strategies that could be developed by the driver so that the truck can cross the desert. Then ask them to determine the most efficient strategy, that is, the one that uses up the least amount of fuel.

Clearly fuel deposits need to be made at intermediate depots in the desert, but where should these depots be placed?

Since each drum can take the truck 100 km, the initial options seem to be to have depots at the 100 km, 200 km and 300 km marks. Note that placing a depot beyond the 300 mark is not an option since no drums could be deposited at this depot and still allow the truck to return to base to collect more fuel. Some of your students may observe that depots could be placed at other marks inside the 300 km mark, and they should be encouraged to investigate these possibilities later. At present you should concentrate for the bulk of the class on the three depots indicated above.

The most efficient options for these depots are shown in Table 1.

Table 1

Depot marks (km)	Drums stored	Drums used	Total
100	5	2	7
200	3	4	7
300	1	6	7

Now placing a depot at the 300 mark suggests an immediate solution. The driver of the truck could complete this action three times until three drums were deposited there, then on the fourth run he or she could refuel at the 300 mark and travel the rest of the way across the desert. This strategy would use 28 drums for a distance travelled of 2800 km. This can be shown diagrammatically as follows in Table 2.

Table 2

Total fuel collected (km)	Out/back usage	300 km depot stored
700	700 → 400	100
	0 ← 300	
1400	700 → 400	200
	0 ← 300	
2100	700 → 400	300
	0 ← 300	
2800	700 → 400	–300 → Cross

Suppose instead that on the first run the driver only uses six drums by depositing two drums at the 200 km mark. The second run would be to use seven drums and deposit one drum at the 300km mark. In the third run the truck can now go right across (see Table 3).

This time only 20 drums were used and the distance covered was 2000 km.

Next ask your students to consider initial deposits at the 100 km mark and develop a strategy from there. They should eventually discover that it is possible to complete the journey using a minimum of 18 drums.

Simpler combinations of the distance to be covered and the truck capacity could be given prior to the above problem so that your students could build up experience with their various strategies. For example, the first problem that your students could be given would be a desert that is 400 km wide with a truck capacity of 3 drums. This is in line with George Polya's (1957) problem-solving technique of solving a simpler, related problem first of all.

Table 3

Total fuel collected	Out/back usage	200 km depot stored	Out/back usage	300 km depot stored
600	600 → 400	200		
	0 ← 200			
1300	700 → 500	200	500 → 400	100
	0 ← 200	200	200 ← 300	
2000	700 → 500	–200	700 → 600	–100 → Cross

A more difficult extension is to find the minimum number of drums needed for a successful trip right across the desert and back, given that there is initially no fuel on the other side of the desert.

An analogous problem is given by Bolt (1989):

The truck has a capacity for 400 km. The amount of fuel at the base is for a trip of 1600 km only. How far into the desert can the truck go and successfully return?

Depots could be tried at the 50, 100,150 and 200 km marks. However a depot at the 200 km mark cannot allow anything to be deposited.

Your students should be asked to investigate the other three marks as first depot possibilities using multiple trips. They should endeavour to use and transport all 1600 km of fuel to that depot as a new launch base, but leave just enough to return to the start at the end. Here are the three possible scenarios for the first depot as the new base.

The 50 km mark has 1250 km deposited with 50 km to be left.

The 100 km mark has 900 km deposited with 100 km to be left.

The 150 km mark has 550 km deposited with 150 km to be left.

When your students consider the latter scenario they should see that the truck can now go only a further 200 km into the desert and then return to the start for a total distance into the desert of 350 km.

Next they should try the 100 km mark scenario. They have 800 km of fuel available and so can establish a second depot at the 200 km mark where they can deposit 500 km of fuel but must leave 100 km for the return trip to the first depot. They can make a further 200 km out from this second depot for a total distance into the desert of 400 km.

Finally they try the 50 km depot. When they try to establish a second depot at the 100 km mark (a further 50 km out) they manage to get 950 km of fuel deposited, but must leave 50 km for the return trip. If they persevere with this approach they cannot even reach 400 km into the desert because they will be forced to leave 100 km of extra fuel at depot 2 because of the truck's limited capacity.

The solution is to set the second depot at a distance so that there will be exactly 800 km of fuel available to establish a third depot further out, and then there will be no wastage of fuel left at the second depot. This is achieved by setting the second depot a distance

$$66\frac{2}{3}\text{ km}\left(=\frac{800}{3}\right)$$

from the 50 km depot. Now a third depot is established at a further 100 km out from the second depot with 500 km of fuel available for a further 200 km trip into the desert and back through each depot picking up the fuel that was left. Ask your students to show this diagrammatically. The total distance is

$$50+66\frac{2}{3}+100+200=416\frac{2}{3}\text{ km}$$

which can be written interestingly as

$$200\left(\frac{1}{4}+\frac{1}{3}+\frac{1}{2}+1\right)$$

Note that the truck's capacity was a factor of the available fuel. Try to figure out a solution when this is not so, as for example if the truck's capacity was 500 km and 1600 km of fuel was still available. Happy discoveries with this one!

References

Bolt, B. (1989). *The mathematical funfair*. Cambridge University Press.

Polya, G. (1957). *How to solve it*. Princeton University Press.