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Things may not always be as they seem: The set shot in AFL football

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"Brown will kick from 30 metres at an angle of about 45 degrees". "Ablett marks deep in the pocket, and swings the ball back to find an unmarked Chapman. Chapman is further from goal than Ablett, but will kick from a better angle".

These pieces of mock commentary illustrate two of the common situations associated with set shots at goal in AFL football. Here we investigate how the difficulty of a "set shot" at goal varies with position on the field.

By *set shot* we mean a player kicking for goal following a mark or free kick. Every such kick will occur from a particular spot, and while a player has some discretion in its choice, our analysis begins with the contact of boot to ball. The two parameters that determine shot difficulty are *angle* and *distance* from the goal, which are fixed for any given set shot. There are choices as to how these are measured, and the definitions used in the analysis that follows have been made because the subsequent mathematics is thereby rendered more tractable. Outcomes of the modelling challenge some accepted folklore, with respect to positions that represent easier or more difficult shots at goal, and these are discussed in a section called implications for practice.

The analysis begins by defining the "angle of opportunity" (θ in Figure 1) as a measure of difficulty for kicks at a given distance and angle from goal, where the "angle of opportunity" is the angle subtended by the goalmouth at the point of contact. The second section explores how different positions on the field provide equal goal scoring opportunity to the kicker, while the third section considers how opportunity changes with distance—for set shots taken directly in front of goal. Contrary to intuition, being straight in front does not necessarily equate to increased goal scoring opportunity—for example, for kicks from 50 metres the angle subtended by the goalmouth is only slightly over 7 degrees. The following three sections first introduce the "equal opportunity angle locus", and then explore its implications. This construct enables the difficulty of shots from different positions on the field to be compared—by equating them respectively with one taken at a particular distance directly in front of goal. For each situation explored, spreadsheet output illustrates practical implications that emerge—outcomes have strategic importance, as well as providing further challenges to conventional wisdom. The analysis to this point has used actual goal width, but idealised goalposts with height but zero width. We next introduce goalposts of typical width, to test the robustness of earlier results. These remain reliable, except for a very few situations—extremely acute angle shots from close to goal. We then consider implications for practice from the viewpoint of coaches, players, and commentators. The paper concludes with a linking of the mathematics and modelling involved, to educational settings in terms of curriculum content and possible teaching approaches.

Angle of opportunity (goal angle)

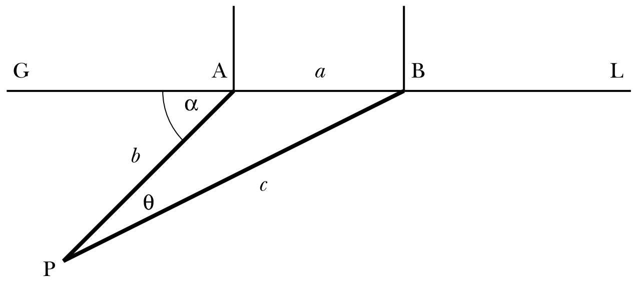

Figure 1. Set shot angle.

We assume a level field, and calm conditions. The precise position of the boundary line is irrelevant, as it serves only to limit how far the kicking point can be from goal is a given direction. Kicking range is assumed to include the extra distance needed to clear defenders on the goal line. When the distance is within kicking range, we can think of the difficulty of a set shot as determined by θ, the angle subtended by the goalmouth at P—the *angle of opportunity* or *goal angle*. In the following:

- P is the point from where the kick is taken;
- GL is the horizontal straight line through the base of the vertical goalposts at A and B;
- AB = a = 6.4 m (distance between AFL goalposts);
- PA = b = distance to near goalpost;
- PB = c = distance to far goalpost;
- ΔAPB is in the horizontal plane of the field, at right angles to the plane containing A, B and the goalposts—or in a parallel plane when P is above the playing surface;
- θ is the angle subtended at P by the goalmouth (AB)—the goal angle.

The position of a kick is fixed by prescribing a distance and an angle; we will use distance from the near post (b), and the angle GAP (α)[1]. (The smaller the angle GAP, the more acute is the shot at goal).

1. This angle leads to simpler mathematics than an angle, measured, for example, from the midpoint of AB.

By the exterior angle theorem $\angle ABP = \alpha - \theta$. (Note this means that $\theta < \alpha$ always.) Applying the sine rule to ΔAPB we have:

$$\frac{a}{\sin\theta} = \frac{b}{\sin(\alpha - \theta)}$$

To find θ it is convenient to expand sin $(\alpha - \theta)$ and rearrange terms to obtain:

$$\tan\theta = \frac{a\sin\alpha}{b + a\cos\alpha} \qquad (1)$$

so

$$\theta = \tan^{-1}\left(\frac{a\sin\alpha}{b + a\cos\alpha}\right) \qquad (2)$$

For example: $b = 30$, $\alpha = 25°$ gives $\theta = 4.32°$
$b = 10$, $\alpha = 45°$ gives $\theta = 17.30°$

Set shots of equal opportunity

For some constant *goal angle* θ (e.g. 10°) we vary α, and calculate the distances b that give equal chances at goal from different angles.

From (1)

$$b = a\left(\frac{\sin\alpha}{\tan\theta} - \cos\alpha\right) \qquad (3)$$

Two sample graphs giving b as a function of α, for fixed values of θ, are shown in Figure 2.

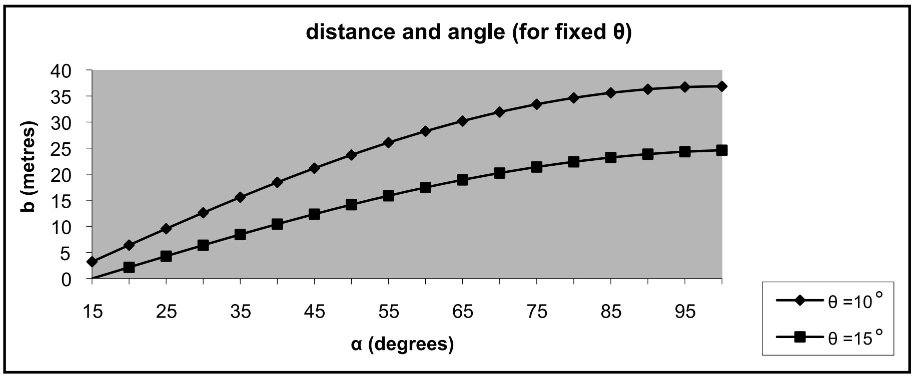

Figure 2. Kicking distance for fixed goal angle.

All points on a given graph give the same "opportunity" as they share the same (constant) value of goal angle θ.

Consider the graph for θ = 10: We see that shots from 10 m at about 25°, and from 30 m at about 65°, give the same "opportunity", as they both have a goal angle of 10°.

Consider the graph for θ = 15: A shot from 10 m at about 40°, has the same "opportunity" as a shot from 20 m at about 70°, as both share the goal angle of 15°.

Set shot directly in front of goal

Next, we look at the situation where the kick is taken directly in front of goal as shown in Figure 3. Here we calculate goal angles for set shots taken directly in front of goal at different distances. It is convenient to use the distance MP = p, rather than b. (However note that b is known when p is specified.)

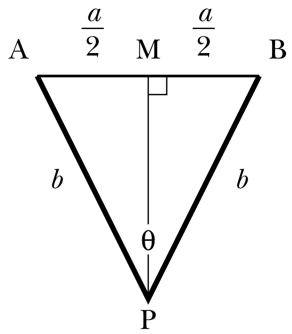

Figure 3. Kicking point P directly in front of goal.

From ΔAMP in Figure 3, we have

$$\tan\frac{\theta}{2} = \frac{a}{2p}$$

$$\theta = 2\tan^{-1}\left(\frac{a}{2p}\right) \qquad (4)$$

For example if $p = 50$ (kicking from 50 m), then $\theta = 7.32387$ (approximately 7.32°). A spreadsheet readily calculates θ for individual values of p—say at 5 m intervals, as shown below in Figure 4. We observe that the goal angle "deteriorates" quickly with distance from goal. For example once the distance exceeds about 35 m, the goal angle becomes less than 10°.

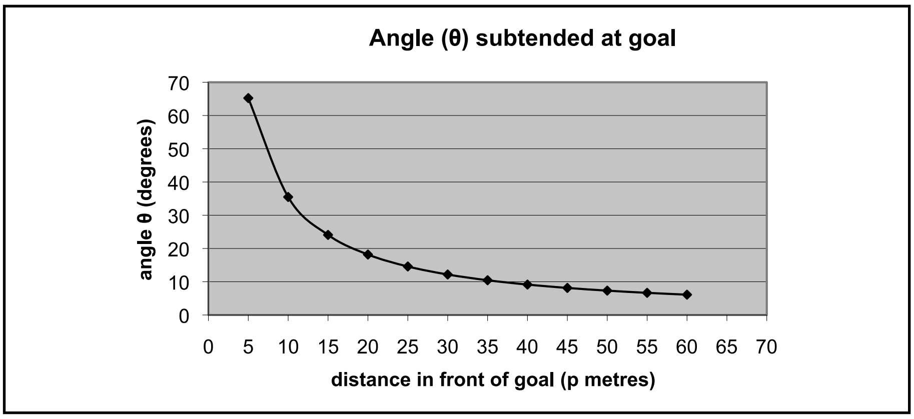

Figure 4. Goal angle for "in front" shot at goal.

Equal opportunity angle locus

Since a circle is defined by three points, for any given point P within the field of play, (assuming a horizontal surface), a circle can be drawn through P and the points A and B where the goal posts intersect the ground as shown in Figure 5. The goal angle θ is then the angle at the circumference of the circle subtended by the goalmouth chord AB. Since AB subtends the same angle at all points on the circumference of the circle, such points represent, as far as goal angle is concerned, kicking points of equal opportunity with respect to a successful attempt at goal.

Furthermore $\angle ACB = 2\theta$ (angle at centre theorem), and $\angle ACR = \angle BCR = \theta$. If the circle has radius r then we have CS = CA = CB = r.

Then $$RC = r\cos\theta$$

and $$D = RS = (RC + CS) = r(1 + \cos\theta)$$

where D is the distance directly in front of goal (at point S), where the goal angle (θ), is the same as at P. (Note that we have a different circle for each different value of θ.)

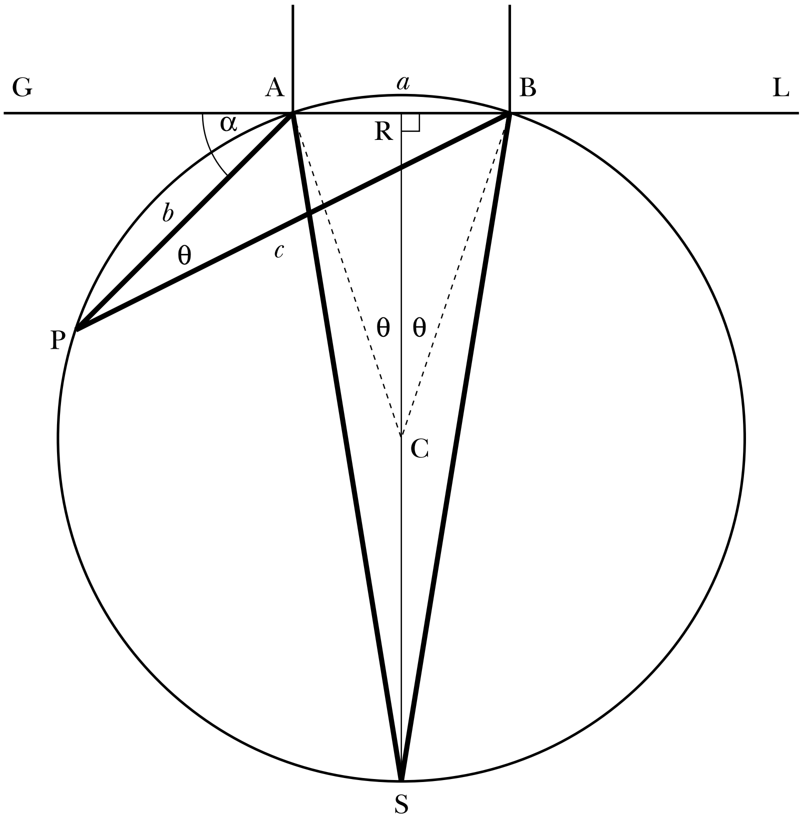

Figure 5. Circle of equal opportunity.

Locating point directly in front of goal with same goal angle as P

By equivalent position we mean the point directly in front of goal that subtends the same goal angle as all other points on the circumference of the circle of equal opportunity (in particular P). We proceed to find the distance (D) directly in front of goal, which affords the same goal angle as the point P. To do this we need to express D in terms of b and α.

From (1) $$\tan\theta = \frac{a\sin\alpha}{b + a\cos\alpha}$$

and we note $$D = \frac{a}{2}\tan\left(\frac{\theta}{2}\right)$$

since $$\angle ASR = \frac{\theta}{2}$$

Since $$\tan\theta = \frac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$$

we obtain $$k\tan^2\left(\frac{\theta}{2}\right)+2\tan\left(\frac{\theta}{2}\right)-k=0$$

where for convenience we put $k=\dfrac{a\sin\alpha}{b+a\cos\alpha}$.

The positive root of this quadratic equation is

$$\tan\left(\frac{\theta}{2}\right)=\frac{\sqrt{\left(k^2+1\right)}-1}{k}$$

which on substituting for k leads to

$$\tan\left(\frac{\theta}{2}\right)=\frac{\sqrt{\left(a^2+b^2+2ab\cos\alpha\right)}-b-a\cos\alpha}{a\sin\alpha}$$

Thus $$D=\frac{0.5a^2\sin a}{\sqrt{\left(a^2+b^2+2ab\cos\alpha\right)}-b-a\cos\alpha} \qquad (5)$$

Since $a = 6.4$, $$D=\frac{20.48\sin\alpha}{\sqrt{\left(40.96+b^2+12.8b\cos\alpha\right)}-b-6.4\cos\alpha}$$

This enables us to calculate the distance directly in front of goal (D) that provides the same goal opportunity as any point P in the field of play, given only the distance (b), and angle (α), that defines P.

For example with $b = 10$:

$$D=\frac{20.48\sin\alpha}{\sqrt{\left(140.96+128\cos\alpha\right)}-10-6.4\cos\alpha}$$

If $\alpha = 25°$ then $D = 37.66$, so a set shot from10 m at an angle of 25° to the goal line has the same goal angle as a shot from directly in front from a distance of about 38 metres. Calculating D for different values of α leads to the curve labelled ($b = 10$) in Figure 6.

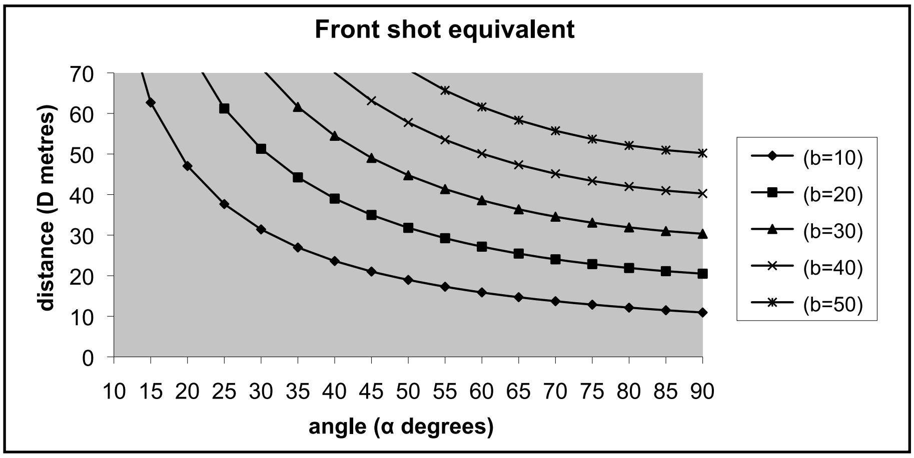

Figure 6. Front shot equivalent distance.

Similarly the other curves give corresponding information for shots taken at various angles from 20, 30, 40, and 50 metres respectively. Thus when $\alpha = 30°$, a shot from 20 m from the near goalpost has the same goal angle as a shot directly in front from a distance of 50 m (using the (b = 20) curve). When $\alpha = 50°$, a shot from 50 m from the near goalpost has the same goal angle as a shot from directly in front from about 70 m—a better prospect for most kickers.

Finding locations with same goal angle as a given point in front of goal

From (4), the goal angles for shots taken from directly in front of goal at distances of 30 m, 40 m, and 50 m, are respectively $\theta = 12.177°$, $\theta = 9.148°$, and $\theta = 7.324°$.

Using (3) we calculate positions of equal opportunity by calculating b, for different values of α, for these fixed values of the goal angle (see Figure 7).

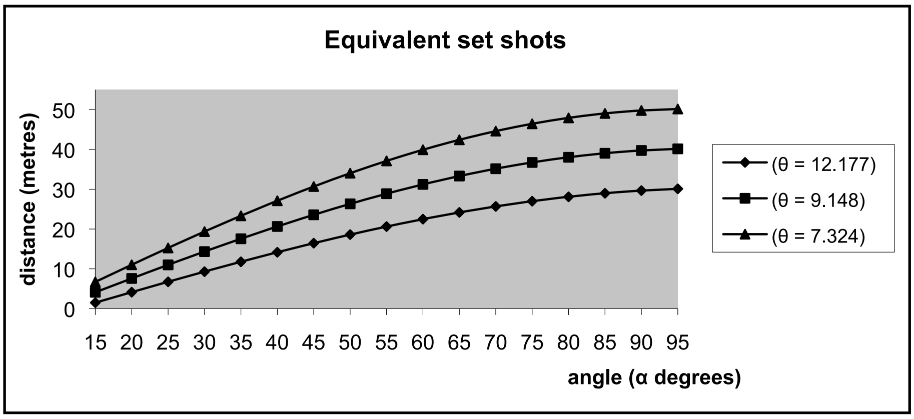

Figure 7. Positions for set shots of equal opportunity.

The distances 30 m, 40 m, and 50 m, are chosen as representative. Other realistic kicking distances can of course be added. These graphs are similar to those in Figure 2—except here the three chosen distances have been used to generate the goal angles used.

All points on the graph ($\theta = 7.324$) have this value as the goal angle. That is, the same goal angle as a set shot from 50 m, directly in front of goal. For example shots from 30 m (approximately) at an angle of 45°, and from 20 m (approximately) at an angle of 30°, have the same opportunity of success as a shot from directly in front at a distance of 50 m, as their points lie on the curve. (More exact distances are 30.7 m and 19.4 m.)

The extreme case for a goal angle of zero, applies to a set shot from outside the boundary line, where the kicker sees only a single post. Options then are to pick out another player, or try to create some angle in the kicking motion, without arousing umpire suspicions that the kicker has played on.

Goalposts with thickness

Here we examine how the analysis is affected when goalposts of given diameter are introduced. As successful kicks must clear defenders on the goal line the height is such that the protective padding around the lower part of the post is irrelevant. Hence the analysis proceeds with standard post diameters.

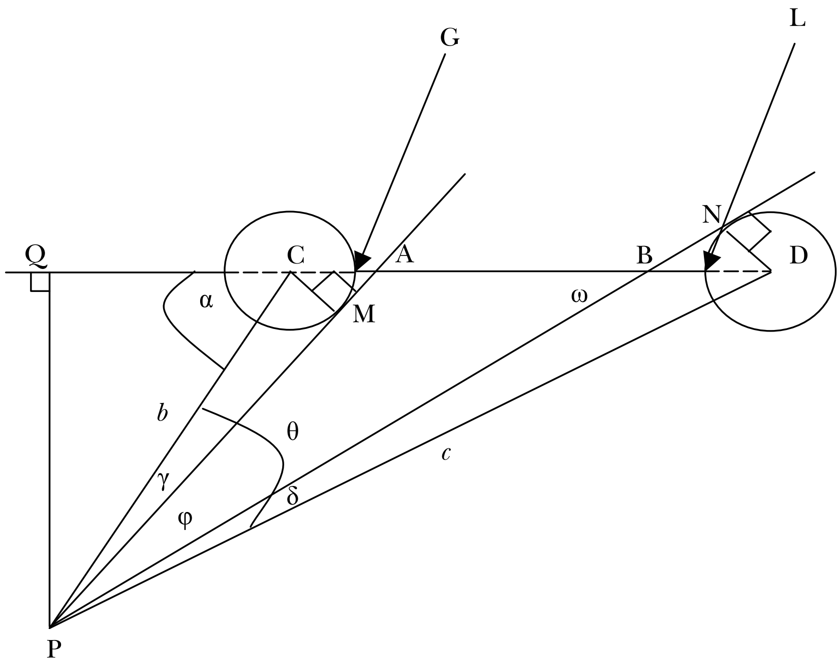

Figure 8. Set shot with cylindrical goal posts.

Abel Manufacturing's website (www.abelflag.com.au/sg_afl_gp_details.html) advertises AFL-compliant posts of varying diameters between 8 cm and 12.5 cm. For purposes of calculations that follow, posts with an 8 cm diameter are used.

The diagram is augmented by the following information.

- Radius of goalpost: CM = ND = s (0.04 m)
- Actual goalmouth width: GL = a (6.4 m)
- Distance from P to centre (C) of near goalpost: PC = b
- Distance from P to centre (D) of far goalpost: PD = c
- Angle subtended by CD at P ($\angle$CPD): $\theta = \gamma + \varphi + \delta$
- Effective available goal line: AB = d.

The actual goalmouth width as prescribed in the laws of the game (6.4 m) is the distance measured between points on the inside circumference of the two posts.

The *effective available goal line* (or available goal width), is the line segment defined on GL by the straight lines (PA and PB) that just brush (are tangent to) the posts at M and N.

Angle φ subtended by AB at P (∠APB) is now the 'angle of opportunity', for a successful kick at goal.

Angle of opportunity (φ)

In ΔPCD:
$$\frac{\mathrm{CD}}{\sin\theta}=\frac{\mathrm{CP}}{\sin(\alpha-\theta)}=\frac{\mathrm{PD}}{\sin(180^\circ-\alpha)}$$

Hence
$$\frac{a+2s}{\sin\theta}=\frac{b}{\sin(\alpha-\theta)}=\frac{c}{\sin\alpha}$$

Thus
$$\tan\theta=\frac{(a+2s)\sin\alpha}{b+(a+2s)\cos\alpha}$$

where
$$c=\sqrt{(a+2s)^2+2(a+2s)b\cos\alpha+b^2}$$

In ΔPCM
$$\sin\gamma=\frac{s}{b}$$

In ΔPDN
$$\sin\delta=\frac{s}{c}=\frac{s}{\sqrt{(a+2s)^2+2(a+2s)b\cos\alpha+b^2}}$$

Hence
$$\varphi=\tan^{-1}\left(\frac{(a+2s)\sin\alpha}{b+(a+2s)\cos\alpha}\right)-\sin^{-1}\left(\frac{s}{b}\right)-\sin^{-1}\left(\frac{s}{c}\right)$$

where
$$c=\sqrt{(a+2s)^2+2(a+2s)b\cos\alpha+b^2}$$

Effective available goal line (AB)

$$\mathrm{AB}=\mathrm{CD}-\mathrm{CA}-\mathrm{BD}$$
$$\mathrm{CA}=\frac{\mathrm{CM}}{\sin(\angle\mathrm{CAM})}=\frac{s}{\sin(\alpha-\gamma)}$$
$$\mathrm{BD}=\frac{ND}{\sin(\angle\mathrm{DBN})}=\frac{s}{\sin\omega}=\frac{s}{\sin(\alpha-\gamma-\varphi)}$$

Hence
$$\mathrm{AB}=d=a-s\left(\frac{1}{\sin(\alpha-\gamma)}+\frac{1}{\sin(\alpha-\gamma-\varphi)}-2\right)$$

This gives the reduced available goal width due to the diameter of the posts.

For example, if $b = 5$ and $\alpha = 2°$, we obtain AB = 2.77 (m) and $\varphi = 0.47°$, "impossible angle" would be the usual descriptor. For the three shots described earlier in relation to Figure 7, the respective values of AB are 6.36 m, 6.30 m, and 6.40 m. These represent reductions of 0.81%, 1.62%, and 0.002% from the case where post thickness is ignored.

More generally, Figure 9 shows how the available goal width changes with distance and angle. In this diagram the upper horizontal line represents the actual distance between the posts (6.4 m). The other graphs show the available goal width for kicks of 10, 30, and 50 metres respectively covering the full range of angles from "impossible" to straight in front of goal. Of course for some kicks (e.g., from 30 and 50 metres), the results are irrelevant for small angles, as the boundary line will intervene to prevent such events.

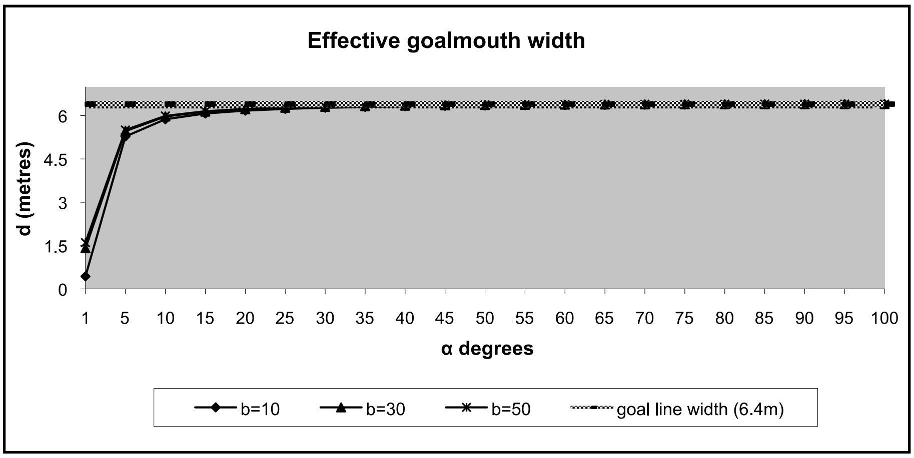

Figure 9. Effective goalmouth width as a function of angle.

Taking account of such practicalities we see that for the vast majority of cases the available goal line differs little from the standard value of 6.4 m; for angles greater than about 15°, the available goal width is over 6 m for any kicking distance. This means that the approximation treating goal posts as one-dimensional provides reliable practical information except for very acute angles. We now proceed to consider implications on this basis.

Some implications for practice

The initial analysis has assumed linear goalposts of zero width, separated by the goal line width of 6.4 metres. We have seen, in the previous section that this does not create distortions except for very acute angles, very close to goal, and we now proceed to consider some implications of the results obtained.

There seems to be one perception that is common to almost everyone, whether player, official, spectator or commentator. This is that the central corridor from goal to goal is the most favourable place to be when kicking for goal. The full view of the goal front afforded from the corridor serves to confirm the perception. What the calculations above show is that the perception is mostly an illusion. For example, when kicking from directly in front of goal from a distance of 50 metres, despite the appearance of generosity, the actual angle, which is available to the kicker, is only a touch above 7°. We have found in the analysis that here are many positions, some at quite substantial angles, which afford better opportunities.

A second perception has to do with the notion that left and right footers are better placed when kicking from an angle from one or other of the forward pockets—and this perception also needs qualification. We have seen, of course, "amazing" goals kicked from the "wrong" side. For a given goal angle it generally does not matter where the kick is taken from: always there is a left post, a right post, and the same fixed angle between them. A kicker is set the same problem of finding a way to steer the ball between the posts whichever foot is used. (Exceptions may occur when the kick is close in from a very acute angle, when the goal angle is impacted substantially by width of posts, and some unusual kicking style is adopted, e.g., a banana kick.)

Some practical implications of the above analyses are suggested below.

Players

The "circles of equal opportunity" show that favourable goal angles occur for many positions other than those in the central corridor. For example, Figure 7 shows that a kick from 30 metres at 45°, has the same goal angle as a kick from 50 metres from directly in front. Striving for greater distance may at times affect kicking technique, so the former may even be a *better* prospect for some players. Further, the tendency for kicks to drift as they lose strength over long distances, provides another argument that shorter and hence stronger kicks, may be preferable other things (goal angle), being equal. While some kicks may follow a curved path their effectiveness is still governed by the requirement to fall inside the goal angle. Appreciation of the possibilities widens the options as to where players may be prepared to lead.

On the other hand, a kick backwards to a player in an assumed "better" position may not provide as much advantage as expected—given also the risk of turning the ball over to the opposition.

Coaches and strategists

The calculations show that the pockets contain many positions as favourable for goal kicking as much of the central corridor. Using these open spaces more expansively makes it harder for defences to man up—so a widely dispersed forward line may also be a more effective one.

There is also the possibility to use circles, or associated markings on the ground, to devise specific targets for goal kicking in practice sessions. Rather than practising kicking goals from a range of generally chosen, and essentially arbitrary positions, the opportunity is there to add precision by selecting points representing specific target locations with respect to distance and angle. Indeed, the opportunity exists to devise specific programs for different individuals (see Figure 10).

Commentators

Observation suggests that commentators in general regard angle, rather than distance, as the main determinant of difficulty. Sometimes there is a comment to the effect that "the distance will test him", but more usually it seems to be the angle that attracts attention. For example, a player who misses a set shot from directly in front of goal, is often given a harder time than one who misses a closer shot from an angle. Yet we have see that such shots may often have the same goal angle, and indeed sometimes the angled shot may be easier when viewed in these terms. Expectations as to what constitutes an easy or difficult shot at goal stand to be modified by an appreciation of goal angle, and the "circle of equal opportunity".

Goal-kicking competitions

Figure 10 depicts in rough form some typical "circles of equal opportunity". Of course, complete circles are unnecessary, as a series of spots on respective circumferences can suffice.

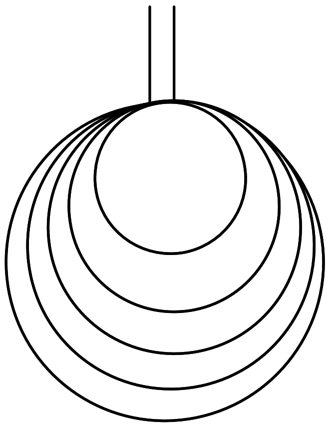

Figure 10. Circles of equal opportunity.

All points on a given circle define the same goal angle, and so are points of equal opportunity for a successful kick (Again, exceptions are positions so adjacent to the goal line that the post configuration impairs the physical ability to view the full angle θ, and the available goal width is reduced).

Various possibilities suggest themselves for the conduct of goal kicking competitions, if a set of circles is drawn on the ground, or a selection of positions defined by such circles marked. One is that contestants be permitted to choose their own kicking position on a given circle. Another is that a second tier be added to the challenge. Following a section where all participants engage in the same set of kicks, add a free choice section where participants select their own positions (analogous to elective dives in Olympic competition). Degrees of difficulty can be assigned according to which circles are selected, and different points awarded on this basis.

Pedagogical options: Some modelling considerations

If set as a general modelling challenge the problem statement might read as follows:

Problem: Investigate how the *difficulty* of a set shot at goal in AFL football *varies with position on the field.*

(Note that the terms in italics are not defined in advance, for defining them through discussion and argument is an essential part of the creation of a mathematical problem from the general real world statement. The substance of this paper illustrates an approach to the general problem—others may be considered).

Assumptions

These have been introduced at various times in the discussion, but the following summary indicates some of the matters that invite specific attention.

- The field is level, and conditions are calm.
- The precise position of the boundary line can be ignored: it simply contributes to a limit on how far P can be from goal.
- A successful kick is determined by:
  (a) sufficient height and length
  (b) appropriate direction,
  so if height and/or length are insufficient the kick fails. So assume that kicks meet necessary requirements of height and length.
- As the precise heights of contact of foot with ball, and of ball at goalmouth are irrelevant, the difficulty (or opportunity) is governed by the size of angle—the problem is two-dimensional. No generality is lost by considering P at ground level in the horizontal plane containing the intersection of the goal posts with the level playing field.
- Whether the kick is straight, curved, or wobbly is irrelevant. Whatever technique is employed, the trajectory of the kick when it reaches the goalmouth, must lie within the angle subtended by the goalmouth at the point of contact.

Strategies for setting up an approach to the problem:

- The kicking point P may be assigned in two main ways:
  (a) by measuring its distance from each goalpost, or
  (b) by specifying a distance (e.g. from one goalpost) and an angle, e.g., as shown in Figure 1.
- The first is convenient if actual measurements can be taken in an empirical approach to the problem.
- The second (used here) is more suited to a generalised analysis, and is consistent with the approach of commentators and players who usually think and talk is terms of (estimated) distance and angle.
- The choice of the angle α (Figure 1) enables the angle that determines difficulty (θ) to be treated as a direct variable in the analysis. (This is not the case if the angle is drawn, say, from the midpoint of the goal line.)

Mathematical concepts and skills

Different sub-problems can be specified if a general modelling approach is not feasible. These are implied within the different sections of the development provided. The following list (not exhaustive) summarises concepts and skills that are invoked within different sections of the analysis:

- exterior angle of triangle theorem
- a circle can be drawn through 3 points
- angle at centre of a circle theorem
- angles in the same segment of a circle theorem
- right-angled triangle trigonometry
- sine rule
- compound angle formula for sin (A–B)
- double angle formula for tangent;
- quadratic equations;
- transformation of formulae
- opportunity for use of CAS
- facility with spreadsheets, including charts.

Design of specific sub-problems will normally depend upon which mathematical topics are under consideration, and/or which techniques are available. In this respect it is useful to consider the substance of senior mathematics curricula. Some of the above mathematics for example, involves the use of the sine rule, within which a form of an expansion like sin (A–B), together with angle theorems, need to be employed, with outcomes displayed on a spreadsheet—and contextualised within an approach associated with the solution of a real world problem, involving a practical setting. Syllabuses for specific subjects do not tend to combine such different elements specifically. However state syllabuses at senior level, and the proposed national curriculum, include them separately as content or approach within a variety of courses, and their combination provides opportunity both to motivate, undertake, and consolidate associated learning. Some relevant curriculum authority websites are included in the references.

References

Abel Manufacturing: http://www.abelflag.com.au/sg_afl_gp_details.html

Australian Curriculum Assessment and Reporting Authority: http://www.australiancurriculum.edu.au/Documents

Queensland Studies Authority: https://www.qsa.qld.edu.au/downloads/senior/snr_maths_b_08_syll.pdf

Victorian Curriculum and Assessment Authority: http://www.vcaa.vic.edu.au/vce/studies/mathematics/mathsstd.pdf